D 92978 (Pages : 2) Name...

Reg. No.....................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3C 03—PROBABILITY THEORY

Time : Two Hours Maximum : 60 Marks

Use of calculator and statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define (i) Mutually exclusive ; (ii) Exhaustive events.
2. Define the terms (i) sample space ; (ii) event.
3. What is the statistical definition of probability ?
4. Given P (A) = 0.6, P (B) = 0.5, P $(A \cup B)$ = 0.8, find P $(A \cap B)$.
5. If A and B are independent events with respective probabilities 0.4 and 0.3, find the probability of at least one of these events to occur.
6. If the random variable X has possible values –1, 0, 3 and 5 with corresponding probabilities 0.2, 0.3, 0.4 and 0.1. Find E (X).
7. If the p.m.f. of X, $f(x) = k$, for $x = 1, 2, 3$ and 4 ; obtain the value of k.
8. Find P (X < 3), if the p.m.f., of X, $f(x) = (2x - 3)/4$, $x = 2, 3$.
9. Define Independent events.
10. If Z is a standard normal random variable, find P (Z < 2).
11. Define Bernoulli distribution.
12. Define Standard Uniform Distribution.

(8 × 3 = 24 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Write the sample space of a random experiment of tossing of two unbiased dice with numbers 1 to 6 written on their faces. Obtain the probabilities of (i) sum of the numbers shown is greater than 10 ; (ii) less than 3.
14. Define conditional probability. If P (A) = 0.4, P (B) = 0.8, P (A/B) = 0.5. Find P $(A \cup B)$.

15. If the random variable X has possible values 1, 2, 3 and 4 with corresponding probabilities 0.2, a, 0.4, b. Find a and b if E (X) = 2.6.

16. Check whether the following is a probability mass function :

    (i) $f(x) = \frac{x-3}{4}$, for $x = 1, 2, 3, 4$. (ii) $f(x) = \frac{x^2}{14}$, for $x = 1, 2, 3$.

17. 20 students are appearing an examination independently each with probability of success 0.60. If X denotes the number students pass in this examination, write the probability mass function of X. Also find (i) the expected number of students to pass the examination ; (ii) probability for all to pass the examination.

18. Define Poisson distribution and state its properties.

19. If X denotes the number of faulty items in a packet of 100 items produced by a good firm. If the average number of faulty items per packet is 2, find the probability that a packet to contain (i) No faulty items ; (ii) at least 2 faulty items.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. X is a random variable with p.m.f, $f(x) = \frac{2x+k}{20}$, for $x = 0, 1, 2, 3$. (i) find k ; (ii) find P (X < 2) ; (iii) $P(X^2 > 3)$; (iv) P(X < 3 / X > 1).

21. Define Normal distribution. State the properties of normal distribution. If X follow normal distribution with mean 10 and variance 9, find (i) P(X < 13) ; (ii) P(X > 7).

(1 × 11 = 11 marks)

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THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3C 02—PROBABILITY DISTRIBUTIONS AND PARAMETRIC TESTS

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the mean and variance of X following binomial distribution with parameters 10 and 0.4.
2. Write down the probability mass function of Poisson distribution.
3. If X follow normal distribution with mean 5 and variance 4. What is $P(X < 5)$?
4. If Z follow standard normal distribution, identify $P(-1 < Z < +1)$.
5. Name any two methods of random sampling.
6. Define cluster sampling.
7. Define critical region.
8. If the P (type II error) in a testing procedure is 0.25, identify the power of the test.
9. Define type I error.
10. Define standard error. Name the probability distribution employed in large sample tests.
11. Which is the test statistic used in large sample testing of population proportion ?
12. A random sample of size 12 is taken from a normal population. To test whether the variance of the population is 4. If the sample variance is 6, find the value of the test statistic.

($8 \times 3 = 24$ marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. 10 students are attending a test with a probability of qualifying the test 0.7, which assumed same for all. Out of these find the probabilities of :

    (i) None to qualify the test. (ii) At least 8 to qualify the test.

14. For a Poisson random variable X, given that $P(X = 0) = e^{-4}$. Obtain the mean and standard deviation of X. Also find $P(X < 2)$.

Turn over

15. If the score of a test is normally distributed with mean 80 and variance 100. What percentage of students, would you expect to have a score more than 60?

16 Explain the method of systematic sampling.

17. The average time to recovery from a disease using a particular medicine is 10 days with a variance of 4 days. Modification has been done one the medicine under the expectation that the recovery time may reduce. In a sample of size 225 with the modified medicine found the average recovery time 8 days. Can we assume that the modified medicine is effective to reduce the recovery time with a 5% significance level ?

18. In a sample of 400 children from a particular community, 24 are found to be with behavior disorder. Test the hypothesis that the true proportion of children with behavior disorder in that community is 0.08 under 5% level of significance.

19. A random sample of 10 boys had the following I.Q's, 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Assuming the population is normal; do these data support the assumption that the populations mean I.Q. of 100 ?

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Explain the large sample test of equality of means of two populations when population variances are (i) known ; (ii) unknown.

(b) Also explain the method of testing equality of proportions of two populations.

21. The scores obtained by students randomly selected from two geographical areas A and B in a particular test are :

From area A	23	26	18	38	25	17	30	19	24		
From area B	28	16	15	27	35	27	24	29	26	18	29

Test whether the mean score differ significantly for the students from area A and B at 5% level of significance.

(1 × 11 = 11 marks)

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THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(Multiple Choice Questions for SDE Candidates)

1. Sum of n independent Bernoulli random variables follows :

   (A) Binomial distribution.
   (B) Poisson distribution.
   (C) Bernoulli distribution.
   (D) Geometric distribution.

2. The distribution that can be used to model number of success in n independent trials with constant probability of success is :

   (A) Binomial distribution.
   (B) Poisson distribution.
   (C) Bernoulli distribution.
   (D) Negative binomial distribution.

3. The moment generating function of a Poisson random variable X is $e^{\left(\frac{e^t-1}{3}\right)}$. Then, X follows :

   (A) $P\left(\frac{1}{3}\right)$.
   (B) P(3).
   (C) $P\left(\frac{2}{3}\right)$.
   (D) $P\left(\frac{3}{2}\right)$.

4. The distribution which can be used for modeling rare events is :

   (A) Binomial distribution.
   (B) Poisson distribution.
   (C) Geometric distribution.
   (D) Discrete uniform distribution.

5. Mode of $B\left(11, \frac{1}{3}\right)$ is at :

   (A) 3.
   (B) 4.
   (C) 4 and 5.
   (D) 3 and 4.

6. Mean of a Poisson random variable with double modes at $x = 5$ and $x = 6$ is :

   (A) 4.
   (B) 5.
   (C) 6.
   (D) 7.

7. If X and Y are independent Poisson random variables with mean 3 and 5 respectively, then the variance of X + Y is :

   (A) 2.
   (B) 3.
   (C) 5.
   (D) 8.

8. If X and Y are independent Poisson random variables with standard deviation 1 and 2 respectively, then the variance of X + Y is :

   (A) 3.
   (B) 9.
   (C) 5.
   (D) 25.

9. The distribution that can be used to model number of trials required to get r success in independent trials with constant probability of success is :

   (A) Binomial distribution.
   (B) Poisson distribution.
   (C) Bernoulli distribution.
   (D) Negative binomial distribution.

10. Mean and variance of U (2,4) are :

   (A) 1 and $\frac{1}{3}$.
   (B) 3 and $\frac{1}{3}$.
   (C) 1 and 3.
   (D) 3 and 3.

11. The sum of n independent exponential random variables follows :

   (A) Exponential distribution.
   (B) Gamma distribution.
   (C) Normal distribution.
   (D) Uniform distribution.

12. The moment generating function of a gamma distribution is $(1-2t)^{-3}$. Then its mean is :

   (A) 12.
   (B) $\frac{1}{12}$.
   (C) 6.
   (D) $\frac{1}{6}$.

13. If $f_x(x) = 2e^{-2x}, 0 < x < \infty$, then E (X) is equal to :

   (A) 2.
   (B) $\frac{1}{2}$.
   (C) 4.
   (D) $\frac{1}{4}$.

14. Normal distribution is symmetric about :

(A) Mean. (B) Median.

(C) Mode. (D) All the above.

15. Moment generating function of N (0, 2) is :

(A) e^{t^2}. (B) $e^{\frac{t^2}{2}}$.

(C) e^{-2t^2}. (D) e^{-t^2}.

16. In a standard normal curve, the area between –2 and +2 is :

(A) 0.4772. (B) 0.9544.

(C) 0.0456. (D) 0.4544.

17. If X ~ N(0, 1) and $P(|X| < 1.13) = 0.74$, then $P(-1.13 < X < 0)$ is :

(A) 0.37. (B) 0.74.

(C) 0.26. (D) 0.63.

18. Chevychev's inequality states that :

(A) $P(|X-\mu| \le \varepsilon) \ge 1-\frac{\sigma^2}{\varepsilon^2}$. (B) $P(|X-\mu| \ge k\sigma) \le \frac{1}{k^2}$.

(C) Both (A) and (B). (D) Neither (A) nor (B).

19. Central limit theorem for a sequence of i.i.d. Bernoulli random variables is given by :

(A) De-Moivre Feller Theorem. (B) Lindeberg Feller Theorem.

(C) Lindeberg Levy's Theorem. (D) De-Moivre's-Laplace Theorem.

20. Central limit theorem for any sequence of i.i.d. random variables is given by :

(A) De-Moivre Feller Theorem. (B) Lindeberg Feller Theorem.

(C) Lindeberg-Levy's Theorem. (D) De-Moivre's-Laplace Theorem.

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THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Obtain the m.g.f. of the random variable X following Bernoulli distribution with parameter p.

2. If the mean and variance of a binomial random variable X are 5 and 4, find P (X = 2).

3. Obtain the variance of X following uniform distribution over $[a, b]$.

4. Find the standard deviation of X following normal distribution with fourth central moment 243.

5. Define parameter and statistic.

6. Define convergence in probability.

7. State Bernoulli's Law of Large Numbers.

8. Define primary and secondary data.

9. In simple random sampling of n units from a population of N units, prove that the probability for each of the items to be included in the sample is n/N.

10. Identify the probability distribution of the mean of a sample of size 36 taken from a normal population with mean 10 and variance 9. Find the probability that the sample mean is greater than 10.5.

11. If X and Y are independent standard normal random variables, identify the probability distributions of (i) X / Y ; (ii) $[X / Y]^2$.

12. Define F-distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If the $(r-1)^{th}$, r^{th}, and $(r+1)^{th}$ central moments of X following Poisson distribution with parameter λ are, μ_{r-1}, μ_r and μ_{r+1}, show that $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda \frac{d}{d\lambda}\mu_r$.

14. If the r.v., X follow uniform distribution over [0,1], show that $Y = -2 \log_e X$ follow exponential distribution.

15. State Chebyshev's inequality. Let X denotes the number shown, when an unbiased die is thrown. Use Chebyshev's inequality to show that $P(|X - 3.5| < 2) > \frac{13}{48}$.

16. State and prove Weak Law of Large numbers.

17. Using Central limit theorem, prove that binomial distribution with parameters (n, p) tends to normal distribution for large n.

18. Explain stratified random sampling.

19. A random sample of size 12 is taken from a normal population with variance σ^2. If the sample variance is 9, identify two numbers a and b such that $P(a < \sigma^2 < b) = 0.80$.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Find the m.g.f. of the random variable X following $N(\mu, \sigma^2)$.

    (ii) If $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$, where X_1 and X_2 are independent, obtain the probability distribution of $Y = aX_1 + bX_2$ for any two constants a and b.

21. (i) Define t-distribution. If t follow $t_{(n)}$, show that $E(t) = 0$.

    (ii) If t follow $t_{(n)}$, prove that t^2 follow $F_{(1,n)}$.

(1 × 11 = 11 marks)

D 92975 **(Pages : 2)** Name.....................................

Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3B 03—STATISTICAL ESTIMATION

Time : Two Hours and a Half Maximum : 80 Marks

Use of calculator and statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find E (X), if X follow continuous uniform distribution over [0, 10].
2. Find P (X > 2), if X is an exponential random variable with mean 0.25.
3. If X and Y are independent exponential random variables with parameter $\lambda = 2$, identify the distribution of X + Y.
4. Define Pareto distribution. State any one of its uses.
5. If X and Y are independent random variables following normal distribution with means 10, 12 and variances 9, 16 respectively. Identify the probability distribution of Z = 3X – 2Y.
6. A random sample of size 25 is taken from a normal population with mean 10 and variance 16. Identify the probability that the sample mean is less than 10.
7. Using the statistical table, find the value of a such that (i) $P\left(\chi^2_{12} < a\right) = 0.05$; (ii) $P(t_{12} < a) = 0.05$ . .
8. Identify the probability distribution of the ratio of two independent standard normal random samples.
9. Differentiate point and interval estimation.
10. Define consistent estimator. What are the sufficient conditions for an estimator t to become consistent for a parameter θ ?
11. State Fisher-Neymaan factorization theorem.
12. Define maximum likelihood estimator.
13. Find the moment estimator of λ, using random sample of size n taken from an exponential population with p.d.f. $f(x) = \lambda e^{-\lambda x}, x > 0$.
14. Define confidence coefficient.
15. Write the test statistic and its sampling distribution to find the confidence interval of the mean of a normal population using small sample and when population variance is unknown.

(10 × 3 = 30 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find the value of a, if X is a random variable following rectangular distribution over $[-a, a]$, where $P(|X| < 2) = P(|X| > 2)$.

17. State and prove the lack of memory property of exponential distribution.

18. Find the mean and variance of X following beta distribution of first kind.

19. If X follow standard normal distribution, show that X^2 follow Chi-square distribution with 1 d.f.

20. The variance of a random sample of size 10 taken from a normal population with mean 6 is 7.21. Calculate the probability that the mean of this sample is less than 5.

21. For the distribution with p.d.f., $f(x) = \lambda e^{-\lambda x}, 0 < x < 1, \lambda > 0$, obtain a sufficient estimator for λ.

22. Explain Maximum Likelihood Estimation. Point out the properties of a MLE.

23. Provide a 95% confidence interval for the mean of a normal population with variance 9 based on 16 samples with sample mean 12.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Define Gamma distribution with two parameters a and p. Find the m.g.f. of a random variable X following this distribution. Also state and prove the additive property of gamma distribution with two parameters.

25. (i) If X follow $N(\mu, \sigma^2)$, find k if $P(X \le k) = 4P(X \ge k)$.

    (ii) Prove that the quartile deviation of normal distribution $N(\mu, \sigma^2)$, is 0.68 times the standard deviation.

26. Define t-distribution. Derive any one of the statistics following t-distribution. If t follow t distribution with n degrees of freedom, show that t^2 follows F-distribution with (l, n) degrees of freedom.

27. Derive the confidence interval for the variance of a normal population $N(\mu, \sigma^2)$ (i) when μ is unknown ; (ii) when μ is known.

(2 × 10 = 20 marks)

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THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Statistics

SG 3C 03—PROBABILITY

Time : Three Hours Maximum : 80 Marks

Section A

Answer all the **ten** *questions.*
Each question carries 1 *mark.*

1. If the events A and B are mutually exclusive, then $P(A \cup B) =$ ——————.

2. Let $F(x)$ be the distribution function of a random variable X. Then $F(-\infty) =$ ——————.

3. $E(X-k)^2$ is minimum when $k =$ ——————.

4. The variance of the Poisson distribution is 6. Then its mean is ——————.

5. If X and Y are independent Poisson variates, the conditional distribution of X given X + Y is ——————.

6. The skewness in a Binomial distribution will be zero if $p =$ ——————.

7. All odd order central moment of —————— distribution are zero.

8. For any *two* events A and B, $P(\bar{A} \cap B)$ ——————.

9. A random variable X has probability distribution as follows :

X	-1	0	1	2
$P(x)$:	$3c$	$2c$	0.4	0.1

The value of constant c is ——————.

10. The difference of two independent normal variates is ——————.

(10 × 1 = 10 marks)

Turn over

Section B

Answer all the **seven** *questions.*
Each question carries 2 *marks.*

11. Define sample space. From an urn containing 4 balles of different colors red (R), blue (B), yellow (Y) and white (W) if two balles are drawn in succession without replacement what is the sample space of this experiment ?

12. Let us consider the random experiment of drawing a card from a pack of cards. What is the probability that the card drawn is a king ?

13. What is the moment generating function of Binomial distribution with parameters $n = 9$ and $p = 1/3$.

14. Define sigma field and probability space.

15. Define random experiment. Give an example.

16. Explain Statistical definition of probability.

17. Define Bernoulli distribution. What is the mean and variance of this distribution ?

(7 × 2 = 14 marks)

Section C

Write any **three** *of the following.*
Each question carries 4 *marks.*

18. Explain mathematical definition of probability. What are its limitations ?

19. Determine the binomial distribution for which the mean is 4 and variance 3. What are the properties of binomial distribution ?

20. Four cards are drawn at random from a pack of 52 cards. Find the probability that (i) They are a king, a queen, a jack and an ace ; and (ii) 2 are kings and 2 are queens.

21. A random variable X has probability distribution as follows :

X	0	1	2	3
$P(x)$	$2k$	$3k$	$13k$	$2k$

Find k and $P(X < 2)$.

22. Distinguish between discrete and continuous random variables. Give example.

(3 × 4 = 12 marks)

Section D

Write any **four** *of the following.*
Each question carries 6 marks.

23. Define binomial distribution. Find the mean and variance of binomial distribution with parameters n and p.

24. If $P(x) = x/15$ if $x = 1, 2, 3, 4, 5$ and 0 otherwise. Find (i) $P(X = 1 \text{ or } 2)$; and

    (ii) $P\left(\frac{1}{2} < X < 5/2 / X > 1\right)$.

25. State and prove addition theorem for two events. Also state the theorem for 3 events.

26. Define normal distribution. Explain the properties of normal distribution.

27. Let X be a random variable with probability mass function :

X	0	1	2	3
$P(x)$	1/3	1/2	1/24	1/8

    Find the expected value of $(X - 1)^3$.

28. For any *two* events A and B prove that $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

(4 × 6 = 24 marks)

Section E

Write any **two** *of the following.*
Each question carries 10 marks.

29. Define expectation of a random variable. State and prove any *two* properties of expectation.

30. What is Poisson distribution ? Give some examples of occurrences of Poisson distribution in different fields. Obtain the moment generating function of Poisson distribution and hence find its mean and variance.

31. Define conditional probability. Give example. For any *three* events A, B and C prove that $P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$.

32. A random variable X has the following probability function :

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k ; (ii) Evaluate $P(X<6)$; (iii) $P(X \geq 6)$; and (iv) $P(0<X<5)$ and the distribution function of X.

(2 × 10 = 20 marks)

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THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Statistics

STS 3C 03—STATISTICAL INFERENCE

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one** *word.*
Each question carries 1 *mark.*

Name the following :

1. Process of estimating of a particular value for a parameter based on the samples taken from that population.
2. The rejection region in testing of hypothesis.
3. The distribution used in small sample testing of equality of variances.

Fill up the blanks :

4. Probability of type I error is called ———— of a test.
5. Probability distribution of a statistics is called its ————.
6. If X_1 and X_2 are two independent standard normal variables, then $t = X_1^2 + X_2^2$ follows ————.
7. If $t \sim t_{(n)}$, then E (t) = ————.

Write True *or* False :

8. If $t_n \xrightarrow{p} \theta$, then t_n is a consistent estimator of θ.
9. Random variable following F-distribution ranges in between $(-\infty, +\infty)$.
10. A statistical hypothesis which is not completely specifies the population is composite hypothesis.

(10 × 1 = 10 marks)

Section B

Answer **all** *questions in* **one** *sentence each.*
Each question carries 2 *marks.*

11. Define statistic.
12. Define interval estimation.

13. Obtain the mean of a chi square random variable with n degrees of freedom.

14. Define size and power of a test in testing of hypothesis.

15. Define most powerful test.

16. State Fisher-Neymaan factorization theorem.

17. What is meant by paired t-test.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. Explain the desirable properties of an estimator.

19. Show that the mode of chi-square distribution with n degrees of freedom is $(n-2)$.

20. Derive a statistic following F-distribution.

21. Obtain the MLE of a and b using samples $x_1, x_2, \ldots x_n$ taken from a uniform (a,b) population.

22. Define type I and type II errors in testing of hypothesis.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. For a random variable of size 16 from $N(\mu,\sigma)$ population, the sample variance is 16. Find a and b such that $P\left(a < \sigma^2 < b\right) = 0.60$.

24. If t follows t-distribution with n degrees of freedom, prove that $Y = t^2$ follows $F(1, n)$.

25. Explain the methods of MLE and the method of moments for identifying an estimator for a parameter.

26. Derive the confidence interval for the mean of a normal population when population standard deviation is : (i) known ; and (ii) unknown.

27. In a sample of 160 items, 24 are damaged. Construct a 99% confidence interval for the true proportion of damaged items.

28. Explain the method of chi square test of variance of a normal population.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. If $X \sim F(n_1, n_2)$ prove that $Y = \frac{1}{X}$ follows $F(n_1, n_2)$. Also find the expectation of X.

30. x_1, x_2 are two random sample taken from a population with pdf $f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, 0 < x < \infty ; \theta > 0$. To test $\theta = 2$ against $\theta = 4$, the critical region is $x_1 + x_2 \geq 9.5$. Obtain the significance level and power of the test.

31. Explain chi-square test of independence. Following table gives the gender and education level of 100 individuals from a village.

Gender\ Education	Primary	High school	College	Total
Male	10	15	25	50
Female	25	10	15	50
Total	35	25	40	100

Test whether Gender and education level are independent.

32. (i) Explain the large sample method of testing the equality of means of two normal populations.

(ii) 40 light bulbs each from two companies are tested and the samples from the first company showed an average life length of 647 hrs with standard deviation 27. Samples from the second company showed an average life length of 638 hrs with standard deviation 31. Test whether the light bulbs from both the companies are having the same life length at 5% level of significance.

(2 × 10 = 20 marks)

D 91720 **(Pages : 3)** Name..................................

Reg. No..................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020

Statistics

STS 3B 03—STATISTICAL ESTIMATION

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one** *word.*
Each question carries 1 *mark*

Name the following :

1. The probability distribution for finding confidence interval for the variance of a normal population.
2. The relative efficiency if t_1 with respect to t_2.
3. The distribution whose importance and application is widened through central limit theorem.

Fill up the blanks :

4. The estimate obtained through the method of minimum variance is called ————.
5. ———— gives the necessary and sufficient condition for an estimate to be sufficient.
6. The reciprocal of an F variate follows ———— distribution.
7. In method of moments, population moments are equated with ————.

Write True *or* False :

8. MLE is at least asymptotically unbiased but not consistent.
9. Efficiency is a desirable property of an interval estimate.
10. For a normal population, the parameters μ and σ are examples of statistic.

(10 × 1 = 10 marks)

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 *marks.*

11. What are distribution parameters ?
12. Define *t* distribution.
13. What do you mean by the sampling distribution of a statistic ?

Turn over

14. Define convergence in probability.

15. Define consistency.

16. State weak law of large numbers.

17. What are the advantages of Chebyshev's Inequality ?

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 *marks.*

18. Give one example each of estimates possessing each of the desirable properties.

19. An unbiased coin is tossed 100 times. Show that the probability that the number of heads will be between 30 & 70 is greater than 0.93.

20. A population is known to follow the normal distribution with mean 2 and standard deviation 3. Find the probability that the mean of a sample of size 16 taken from this population is greater than 2.5.

21. State Cramer- Rao Inequality and explain the method of minimum variance.

22. Obtain the confidence Interval for μ when σ is known.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 *marks.*

23. An unbiased die is thrown 100 times. Find by using Central Limit theorem the probability that the mean of the number thrown will be less than 3.8.

24. A sample of 100 voters were asked to vote in a gallop poll. 55 % of them voted in favour of a candidate. Find 95% & 99% confidence limits for the proportion of voters in favour of the candidate.

25. Find the least number of times a fair coin should be tossed in order that the probability will be at least 0.95 that the frequency ratio of the number of heads differs by less than 0.1 from 0.5.

26. Establish the relation between a t variate and an F variate.

27. Obtain the sampling distribution of nS^2/σ^2 where S^2 is the variance of a sample of size n taken from a Normal Distribution.

28. Obtain the maximum likelihood estimates of the parameters a and b based on a sample taken from a population with uniform distribution in the interval (a, b).

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. 1.5, 2.3, 1.4, 3.6, 2.5 is a random sample from a population. Obtain the mle of θ if the p.d.f. of the population is $f(x) = \frac{1}{2}e^{-[x-\theta]}$; $-\alpha < x < \alpha$

30. State and prove Lindberg Levy form of Central Limit Theorem.

31. Obtain an unbiased estimator of the population variance based on a sample from normal population. Find its value for the following sample 11, 26, 18, 31, 23, 15, 9, 0, 13, 19.

32. Show that $\frac{(\bar{x}-\mu)}{s/\sqrt{n-1}}$ follows t distribution with $n-1$ degrees of freedom where $\bar{x}$ is the mean and S is the standard deviation of a sample taken from Normal population.

(2 × 10 = 20 marks)