D 92463 (Pages : 2) Name..

Reg. No......................................

FIFTH SEMESTER U.G. (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION, NOVEMBER 2020

Mathematics

MAT 5D 19—MATHEMATICS FOR SOCIAL SCIENCES

Time : Two Hours Maximum : 40 Marks

Part A

All questions to be attended.
Each question carries 1 mark.

1. What is an odd function ?
2. What is the derivative of sin x ?
3. What is the solution of $x^2 - 4 = 0$?
4. Define exponential function.
5. Evaluate $\underset{x \to 1}{\text{Lt}}\ x^2 - 1$.
6. What is a linear function ?

(6 × 1 = 6 marks)

Part B

All questions can be attended and overall ceiling.
Answer any **five** *questions.*
Each question carries 2 marks.

7. Solve $2x^2 - 3x + 1 = 0$.
8. State mean value theorem.
9. Find the slope of the curve $ax^2 + bx + c$.
10. What is point of inflection ?
11. If a particle moves along a curve with velocity given by the function $f(x) = 5x^3 + 3x^2 - x + 4$ find the acceleration at $x = 1$.
12. Form the partial differential equation of $z = ax + by + c$.
13. If the marginal revenue function is given by $Q^2 + 12Q - 21$. Find the total revenue function.

(5 × 2 = 10 marks)

Part C

All questions can be attended and overall ceiling.
Answer any **three** *questions.*
Each question carries 4 marks.

14. Evaluate $\int e^{2x} dx$.

15. Find the second partial derivatives of $z = x^3 + y^3 + 3xy$.

16. Find the sum and product of roots of equation $2x^2 + x - 3 = 0$.

17. If $f(x) = \sin x$ and $g(x) = 3x + 1$ find f.g. and g.f.

18. A particle is oving along a curve $S = 12t - 3t^2$. Find the velocity and acceleration at $t = 0$.

(3 × 4 = 12 marks)

Part D

All questions can be attended and overall ceiling.
Answer any two questions.
Each question carries 6 marks.

19. Differentiate x^n using first principle.

20. If $y = a\cos mx + b\sin nx$ show that $d^2y \backslash dx^2 + n^2 y = 0$.

21. Integrate the following : i) log x ; ii) $\left(4x^3 + 2x^2 + x - 3\right) / x^2$.

(2 × 6 = 12 marks)

D 92462 (Pages : 4) Name..................................

Reg. No...............................

FIFTH SEMESTER U.G. DEGREE (SPECIAL) EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5D 18—MATHEMATICS FOR NATURAL SCIENCES

Time : Two Hours Maximum : 40 Marks

Section A

All questions to be attended.
Answer **all** *the six questions.*
Each question carries 1 *mark.*

1. Write the relation between mean, median and mode.

2. Write $\log_b\left(\frac{xy^4}{z^3}\right)$ as the sum or difference of logarithms of x, y and z.

3. Find the harmonic mean H of the numbers 3, 5, 6, 6, 7, 10 and 12.

4. Define the semi-interquartile range.

5. Find the second and third and moments of the set 2, 3, 7, 8, 10.

6. Write the formula for finding skewness of a distribution.

(6 × 1 = 6 marks)

Section B

All questions can be attended and overall ceiling.
Answer any **five** *out of seven questions.*
Each question carries 2 *marks.*

7. Prove that the quadratic mean of two positive unequal numbers a and b is greater than their geometric mean.

8. If $Z_1 = X_1 + Y_1$, $Z_2 = X_2 + Y_2$; ....; $ZN = X_N + Y_N$: prove that $\overline{Z} = \overline{X} + \overline{Y}$.

9. Show that the product of the numbers 5.74 and 3.8, assumed to have three and two significant figures, respectively, cannot be accurate to more than two significant figures.

Turn over

10. The number of ATM transactions per day were recorded at 15 locations in a large city. The data were : 35, 49, 225, 50, 30, 65, 40. 55, 52, 76, 48, 325, 47, 32 and 60. Find : (a) the median number of transactions ; and (b) the mean number of transactions.

11. On a final examination in mathematics, the mean was 72 and the standard deviation was 15. Determine the standard scores (i.e., grades in standard-deviation units) of students receiving the grades : (a) 60 ; (b) 93 ; and (c) 72.

12. Prove that (a) $m_2 = m_2' - m_1'^2$; (b) $m_3 = m_3' - 3m_1' m_2' + 2m_1'^3$.

13. Prove that $w^2 + pw + q$, where p and q are given constants, is a minimum if and only if $w = \frac{-1}{2} p$.

(5 × 2 = 10 marks)

Section C

All questions can be attended and overall ceiling.
Answer any **three** *out of five questions.*
Each question carries 4 marks.

14. Solve the following logarithmic equation ln $(5x)$ – ln $(4x + 2) = 4$.

15. The numbers $X_1, X_2 ...., X_K$ occur with frequencies $f_1, f_2 ..., f_K$ where $f_1 + f_2 + ... f_K = N$ is the total frequency.

    (a) Find the geometric mean G of the numbers.

    (b) Derive an expression for log G.

    (c) How can the results be used to find the geometric mean for data grouped into a frequency distribution ?

16. Given the sets 2, 5, 8, 11, 14 and 2, 8, 14, find :

    (a) The variance of each set.

    (b) The variance of the combined sets.

17. Find the (a) First ; (b) Second, (c) Third ; and (d) Fourth moments about the origin 4 for the set 2, 3, 7, 8, 10.

18. Find the : (a) Quartile ; and (b) Percentile coefficients of skewness for the distribution with $Q_1 = 268.25$, $Q_2 = 279.06$, $Q_3 = 290.75$.

(3 × 4 = 12 marks)

Section D

All questions can be attended and overall ceiling.
Answer any **two** *out of three questions.*
Each question carries 6 marks.

19. The final grades in Mathematics of 80 students at State University are recorded in the accompanying table :

68	84	75	82	68	90	62	88	76	93
73	79	88	73	60	93	71	59	85	75
61	65	75	87	74	62	95	78	63	72
66	78	82	75	94	77	69	74	68	60
96	78	89	61	75	95	60	79	83	71
79	62	67	97	78	85	76	65	71	75
65	80	73	57	88	78	62	76	53	74
86	67	73	81	72	63	76	75	85	77

With reference to this table, find :

(a) The highest grade.

(b) The lowest grade.

(c) The range.

(d) The grades of the five highest-ranking students.

(e) The grades of the five lowest-ranking students.

(f) The grade of the student ranking tenth highest.

(g) The number of students who received grades of 75 or higher.

(h) The number of students who received grades below 85.

(i) The percentage of students who received grades higher than 65 but not higher than 85.

20. Find the mean deviation for the distribution of heights of 100 students in the following table :

Height (in)	*Frequency* (*f*) :
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 – 74	8

21. Calculate the first four moments about the mean for the distribution of the following table :

X	*f*
12	1
14	4
16	6
18	10
20	7
22	2

(2 × 6 = 12 marks)

D 92460–A (Pages : 6) Name....................................

Reg. No....................................

FIFTH SEMESTER U.G. DEGREE (SPECIAL) EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

1. Which of the following is a linear differential equation ?

 (A) $y'' + (y')^2 = \sin x$.
 (B) $(y'')^2 + 3y = e^x$.
 (C) $y'' + 3y' + y = 0$.
 (D) $(y'')^2 + (y')^3 + e^x = 0$.

2. Which of the following is a separable differential equation ?

 (A) $\frac{dy}{dx} = \frac{x^2}{1-y^2}$.
 (B) $\frac{dy}{dx} = \frac{x+y}{x-y}$.
 (C) $\frac{dy}{dx} + (\sin x)\, y = e^x$.
 (D) $\left(\frac{dy}{dx}\right)^2 + (\sin x)\, y = 0$.

3. The general solution of the differential equation $2x\left(3x + y - ye^{-x^2}\right)dx + \left(x^2 + 3y^2 + e^{-x^2}\right)dy = 0$ is :

 (A) $x^2y + ye^{-x^2} + 2x^3 + y^3 = C$.
 (B) $x^2y^2 + ye^{x^2} + 2x + y^2 = C$.
 (C) $xy + ye^{-x^2} + y^2 = C$.
 (D) $xy^2 + y + 2x^3e^{-x^2} + y^3 = C$.

4. Which of the following is an initial value problem ?

 (A) $y' + y = 0, y(0) = y'(0) = 0$.
 (B) $y' + y = 0, y(0) = y(1) = 0$.
 (C) $y'' + y = 0, y(0) = 0, y(1) = 1$..
 (D) $y''' + y = 0, y(0) = 0, y(2) = 4$.

5. Which of the following is a boundary value problem :

 (A) $y' + y = 0, y(0) = 1, y'(0) = 0$.
 (B) $y'' + 5y = 0, y(0) = 1, y'(0) = 3$.
 (C) $x^2y'' + xy' + y = 0, y(0) = 0, y(1) = 2$.
 (D) $y''' + y'' + y = 0, y(0) = y'(0) = y''(0) = 0$.

6. The general solution of the differential equation $y' = \cos x$ is :

(A) $y = \sin x$.

(B) $y = \cos x$.

(C) $y = C \sin x$.

(D) $y = \sin(x) + C$.

7. If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x only, then an integrating factor of $Mdx + Ndy = 0$ is :

(A) $\mu(x) = \exp\left[\int \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) dx\right]$.

(B) $\mu(x) = \exp\left[\int \frac{1}{N}\left(\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}\right) dx\right]$.

(C) $\mu(x) = \int \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) dx$.

(D) $\mu(x) = \int \frac{1}{N}\left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) dx$.

8. If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ is a function of y only, then an integrating factor of the differential equation $Mdx + Ndy = 0$ is :

(A) $\mu(x) = \exp\left[\int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\right]$.

(B) $\mu(x) = \exp\left[\int \frac{1}{M}\left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dy\right]$.

(C) $\mu(x) = \int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy$.

(D) $\mu(x) = \int \frac{1}{M}\left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dy$.

9. An integrating factor of the differential equation $\frac{dx}{dy} + P(x) = Q(x)$ is :

(A) $e^{\int p dx}$.

(B) $e^{-\int p dx}$.

(C) $e^{\int p^2 dx}$.

(D) $e^{\int (P + Q) dx}$.

10. A mathematical model of an object falling in the atmosphere near the surface of earth is given by :

(A) $m\frac{dv}{dt} = mg - rv.$

(B) $m\frac{d^2v}{dt^2} = mg - rv.$

(C) $\frac{dv}{dt} = mg.$

(D) None of these.

11. The differential equation $y'' - 5y' + 6y = 0$ has :

(A) Two linearly independent solutions.

(B) Three linearly independent solutions.

(C) Four linearly independent solutions.

(D) Infinite number of linearly independent solution.

12. The general solution of the differential equation $(D^2 - 4D + 4)\,y = 0$ is :

(A) $(c_0 + c_1x)\,e^{2x}.$

(B) $(c_0 - c_1x)\,e^{2x}.$

(C) $c_1e^x\ c_2e^{-2x}.$

(D) $c_1e^{2x} + c_2e^{2x}.$

13. The characteristic roots of the differential equation $(D^2 - 2D)\,y = 4x^2 + 2x + 3$ are :

(A) $\lambda = 0, \lambda = -2.$

(B) $\lambda = 1, \lambda = 3.$

(C) $\lambda = 0, \lambda = 2.$

(D) $\lambda = 1, \lambda = -2.$

14. The Laplace transform of the unit step function $u_{(t-a)}$ is :

(A) $e^{-as}.$

(B) $e^{-as}/s.$

(C) $e^{as}/s.$

(D) $e^{-as}/s^2.$

15. If $\mathcal{L}\{f(t)\} = \mathrm{F}(s)$, then $\mathcal{L}\{f(at)\} =$

(A) $\frac{1}{a}\mathrm{F}(s/a)$.

(B) $\mathrm{F}(s/a)$.

(C) $\mathrm{F}(a/s)$.

(D) $\mathrm{F}(s)$.

16. $\int_0^\infty \frac{\sin t}{t}\, dt =$

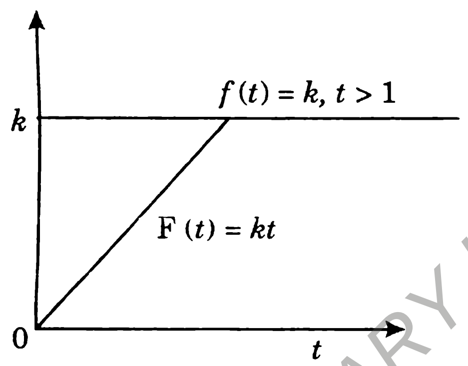
(A) $\frac{\pi}{4}$.

(B) $\frac{\pi}{8}$.

(C) $\frac{\pi}{2}$.

(D) None of these.

17. The Laplace transform of the function whose graph shown below is :



(A) $\frac{k}{s^2}\left(1 - e^{-s}\right)$.

(B) $\frac{k}{s}\left(1 - e^{-s}\right)$.

(C) $\frac{1}{s}\left(1 - e^{-s}\right)$.

(D) None of these.

18. $\mathcal{L}\{\sin hat\} =$

(A) $\frac{a}{s^2 - a^2}$.

(B) $\frac{a}{s^2 + a^2}$.

(C) $\frac{s}{s^2 - a^2}$.

(D) $\frac{s}{s^2 + a^2}$.

19. $\mathcal{L}\left\{t^n\right\} =$

(A) $\frac{n!}{s^n}$.

(B) $\frac{(n+1)!}{s^n}$.

(C) $\frac{n!}{s^n + 1}$.

(D) $\frac{1}{s^n}$.

20. The solution to the problem :

$\alpha^2 u_{xx} = u_{tt}, 0 \le x \le L$

$\begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases}$ $\begin{cases} u(x,0) = f(x) \\ u_t(x,0) = 0 \end{cases}$ is given by

(A) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi t}{L}\right)$.

(B) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi t}{L}\right)\cos\left(\frac{n\pi x}{L}\right)$.

(C) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$.

(D) $u(x,t) = \sum_{n=1}^{\infty} b_n \cos\left(\frac{n\pi x}{L}\right)$.

D 92460 (Pages : 4) Name.................................

Reg. No.................................

FIFTH SEMESTER U.G. DEGREE (SPECIAL) EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Write down the differential equation whose solution is $y = c_1 e^{t} + c_1 e^{-t}$.

2. Find that function given by the Wronkian $W\left[t, t^2\right]$?

3. Find the integrating factor of $\left(x^2 - 1\right)\dfrac{dy}{dx} + 3y = x$?

4. Find the complimentary function corresponding to $y'' + 2y' + y = t$.

5. Compute $\mathcal{L}\left\{e^{\mu t} \cos^2 t\right\}$.

6. What is one dimensional heat equation ?

7. Find $\mathcal{L}^{-1}(1)$.

8. Find the fundamental solutions of $y'' + 25y = t$.

9. Find the value of b_n in the Fourier series expansion of 2π-periodic function $f(x) = x^2, x \in [-\pi, \pi]$.

10. Show that derivative of an even function is odd.

Turn over

11. What do you mean by an exact differential equation ? Give an example.

12. The order of the differential equation $\frac{d^4y}{dx^4} - \left(\frac{dy}{dx}\right)^7 = 10 + \frac{d^5y}{dx^5}$ is ....

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Convert $y'' + y = 0$ into a system of first order equations.

14. Find the Fourier sine series for the 2π-periodic, function $f(x) = -x,\ x \in [-\pi, \pi]$.

15. Find the integrating factor for $ydx - xdy = 0$.

16. Find the inverse Laplace transform of log $((s-a)/(s-b))$.

17. Define unit step function and find its Laplace transform.

18. Solve : $t^2\ y'' - 2ty' - 3y = 0$.

19. Show that the inverse Laplace transform is linear.

20. Write the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.

21. State Abel's theorem.

22. Solve : $\frac{dy}{dx} = (2x - 3y + 1)^2$.

23. Evaluate $\mathcal{L}\left(te^t \sin 2t\right)$.

24. Find the second order p.d.e. for which $u = f(x + at) + g(x - at)$ is a solution.

25. Find $\mathcal{L}\left[\cosh^2 2t\right]$.

26. Solve $y' + 2y = 0$ using Laplace transform.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 marks.
All questions can be attended.
Overall Ceiling 45.

27. Express the function $f(t) = \begin{cases} \sin t, & \text{if } 0 \le t < \pi/2, \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0 & \text{elsewhere.} \end{cases}$ in terms of combination of unit step functions and hence find its Laplace transform.

28. Use method of convolution to find the Laplace inverse of $\frac{1}{(s-1)^3}$.

29. Evaluate the Laplace inverse transforms of $\cot^{-1}(s/a)$ and $\frac{1}{\left(s^2 - 5s + 6\right)^2}$.

30. Derive the Euler formula for finding a_0 in the Fourier expansion of a 2π-periodic function $f(x)$.

31. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

32. Find the Fourier cosine series for the function $f(t) = |t - \pi|, t \in [0, \pi]$.

33. Find the solution of the heat conduction problem : $36u_{xx} = u_t$, $0 < x < 1$, $t > 0$; $u(0, t) = 0$, $u(1, t) = 0$, $t > 0$; $u(x, 0) = \cos(2\pi x) - \cos(5\pi x)$, $0 \le x \le 1$.

34. Get a formula for $\mathcal{L}(f(t))$ where $f(t)$ is a periodic function of period T.

35. State and prove convolution theorem for Laplace transforms.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
Each question carries 15 *marks.*

36. (a) Apply method of variation of parameters to solve : $y'' - y = \sec t$.

(b) Solve $(2x + y + 1)dx + (x - 3y + 2)\,dy = 0$.

37. (a) Solve the following differential equation in two ways, one of them must be using Laplace transform. If $y'' - 2y = t$, $y(0) = 1$, $y(1) = 0$.

(b) Find the Half range Fourier sine series of $f(x) = x^2$, $x \in [0, 3]$.

38. Use method of separation of variables and solve the one dimensional heat equation completely. State the assumptions involved therein explicitly.

(1 × 15 = 15 marks)

D 92459–A (Pages : 4) Name..................................

Reg. No..................................

FIFTH SEMESTER U.G. (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION, NOVEMBER 2020

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. If $A_n = \{n, n+1, n+2, ...\}$, then, $\bigcap_{n=1}^{\infty} A_n = ...$

   (A) 1. (B) $\varnothing$.

   (C) ∞. (D) n.

2. If A = {1, 2, 3} and B = {4, 5} which of the following is not a member of A × B.

   (A) (1,4). (B) (2, 5).

   (C) (3, 4). (D) (4, 3).

3. Which of the following subset of A × A defines a function on $A = \{x \in \mathbb{R} : -1 \le x \le 1\}$.

   (A) $C = \{(x, y) : x^2 + y^2 = 1\}$. (B) $C = \{(x, y) : x + y^2 = 1\}$.

   (C) $C = \{(x, y) : x^2 + y = 1\}$. (D) None of these.

4. Which of the following set is *not* countable ?

   (A) $\{1, 2, ..., n\}$. (B) The set $\mathbb{N}$ of natural numbers.

   (C) The set $\mathbb{Q}$ of rational numbers. (D) The interval (0, 1).

5. The number of injections from $S = \{1, 2\}$ to $T = \{a, b, c\}$ is ———.

   (A) 2. (B) 4.

   (C) 6. (D) 8.

6. If $a \in \mathbb{R}$ such that, $0 \le a < \varepsilon$ for every $\varepsilon > 0$ then, :

   (A) $a > 0$. (B) $a \ne 0$.

   (C) $a = 0$. (D) None of these.

7. The binary representation of 3/8 is, :

   (A) 0.0111111.... (B) 0.0101000.....

   (C) 0.1011111.... (D) 0.0101111.....

8. If $0 < b < 1$, $\lim (b^n)$ equal to :

   (A) 0. (B) 1.

   (C) b. (D) ∞.

9. Limit of the sequence $\left(\frac{3n+2}{2n+1}\right)$ is ———.

   (A) 3. (B) 1/2.

   (C) 2. (D) 3/2.

10. The smallest value of $K(\varepsilon)$ corresponding to $\varepsilon = .01$ for the sequence $\left(\frac{1}{n}\right)$ is ———.

(A) 10. (B) 50.

(C) 100. (D) 101.

11. Which of the following is false ?

(A) If (x_n) is a convergent sequence then $\left(x_n^2\right)$ is convergent.

(B) If (x_n) is a convergent sequence, and $x_n \geq 0$ for every n, then $\left(\sqrt{x_n}\right)$ is convergent.

(C) If $\left(x_n^2\right)$ is a convergent sequence then (x_n) is convergent.

(D) If (x_n) is a convergent sequence then $\left(x_n^3\right)$ is convergent.

12. If $0 < a < b$, then $\lim \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is :

(A) b. (B) a.

(C) $a + b$. (D) ∞.

13. The limit of the sequence defined inductively by, $x_1 = 1$ and $x_{n+1} = 2 + \frac{1}{x_n}$ is :

(A) $1 - \sqrt{2}$. (B) $1 + \sqrt{2}$.

(C) $2 + \sqrt{2}$. (D) $2 - \sqrt{2}$.

14. Which of the following sequences with n^{th} term x_n diverges ?

(A) $x_n = 1 - \frac{(-1)^n}{n}$. (B) $x_n = \frac{1-(-1)^n}{n}$.

(C) $x_n = 1 - (-1)^n + \frac{1}{n}$. (D) $x_n = \frac{(-1)^n (n+1)}{n^2 + 1}$.

15. Which of the following statement is *not* true about closed sets ?

(A) Arbitrary union of closed sets is closed.

(B) Arbitrary intersection of closed sets is closed.

(C) If X = (x_n) is a sequence of elements in a closed set F, then lim X belongs to F.

(D) A subset of R is closed if and only if it contains ail of its cluster points.

16. Which of the following statements is *not* true about Cantor set ?

(A) Cantor set is closed.

(B) Cantor set is uncountable..

(C) he compliment of Cantor set in [0, 1] has length 1.

(D) Cantor set has non-empty open intervals as subsets.

17. If $z = (x, y)$ is a complex number its inverse z^{-1} is :

(A) $\left(\frac{x}{(x^2+y^2)}, \frac{y}{(x^2+y^2)}\right)$.

(B) $\left(\frac{x}{(x^2+y^2)}, \frac{-y}{(x^2+y^2)}\right)$.

(C) $\left(\frac{y}{(x^2+y^2)}, \frac{x}{(x^2+y^2)}\right)$.

(D) $\left(\frac{y}{(x^2+y^2)}, \frac{-x}{(x^2+y^2)}\right)$

18. $(1-i)^4$ is equal to :

(A) 4.

(B) $4i$.

(C) -4.

(D) $-4i$.

19. If = $\frac{-2}{1+\sqrt{3}i}$, then Arg z is :

(A) $\frac{\pi}{3}$.

(B) $-\frac{\pi}{3}$.

(C) $\frac{2\pi}{3}$.

(D) $-\frac{2\pi}{3}$.

20. $\left|e^{i\theta}\right|$ is equal to :

(A) $\sqrt{2}$.

(B) $-\sqrt{2}$.

(C) 1.

(D) -1.

D 92459 (Pages : 3) Name.....................................

Reg. No..................................

FIFTH SEMESTER U.G. (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION, NOVEMBER 2020

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Infimum of the $S = \{1 - 1/m ; m \in N\}$ is ————.

2. The Set of all real numbers which satisfy the inequality $|x^2 - 1| \leq 3$ is ————.

3. Fill in the blanks : The ε neighbourhood of $b \in R$ is ————.

4. State the Supremum Property of R.

5. State Cantor's theorem.

6. Fill in the blanks : If X is a converging sequence of non-negative real numbers, then $\lim X$ = ————.

7. Define Cauchy sequence.

8. State the Betweenness property of irrational numbers.

9. State the Monotone Sub-sequence Theorem.

10. Give an example of a bounded real sequence which is not a Cauchy sequence.

11. Fill in the blanks : The value of $(1+i)^{10} + (1-i)^{10}$ is ————.

12. Fill in the blanks : $\text{Arg}(-\pi - i\pi) =$ ————.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Define Supremum and Infimum of set. Find them for the Set $S = \left\{1 - \frac{(-1)^n}{n} ; n \in N\right\}$.

Turn over

14. Prove that the set N of positive integers is not bounded above.

15. If b is a real number such that $0 \le b \le \varepsilon$ for every $\varepsilon > 0$, then prove that $b = 0$.

16. Show that there doesnt exist a rational number r such that $r^2 = 7$.

17. State and prove Squeeze theorem on sequence.

18. Test the convergence of the sequence $\left(\frac{\log n}{n}\right)$......

19. If X and Y are convergent sequences of real numbers satisfying $(x_n) \le (y_n), \forall n \in \mathrm{N}$, then prove that $\lim(x_n) \le \lim(y_n), \forall n \in \mathrm{N}$.

20. Prove that every bounded sequence of real numbers has a converging sub-sequence.

21. Illustrate Cauchy sequence by an example.

22. Distinguish between converging sequence and Cauchy sequence . Write the relation between them.

23. Prove or disprove that "the union of infinitely many closed sets in R is closed ".

24. Prove that $\left||z_1| - |z_2|\right| \le |z_1 - z_2|$.

25. Test the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.

26. Find the principal value of $(-125i)^{\frac{1}{3}}$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. State and prove Characterization theorem of Intervals .

28. Prove that [0, 1] is uncountable.

29. State and prove the "Density theorem ".

30. Determine the set $\mathrm{A} = \left\{x \in \mathrm{R} : \frac{2x+1}{x+2} < 1\right\}$.

31. Let $\mathrm{X} = (x_n)$ be a non-negative sequence of real numbers with $\lim(x_n) = x$. Prove that $\lim\left(\sqrt{x_n}\right) = x$.

32. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

(c) Justify the property of the sequence (x_n) of positive real numbers with $\lim(x_n)^{\frac{1}{n}} = 1$.

33. Test whether the (x_n) defined by $x_n = 1 + 1/2 + 1/3 + \ldots + 1/n$ is Cauchy sequence or not.

34. Establish the relation between contractive sequence and convergent sequence.

35. Give an algebraic proof for the triangle inequality of complex numbers.

$|(z_1 + z_2)| \leq |(z_1)| + |(z_2)|. \forall z_1, z_2 \in C.$

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) Distinguish between countable and uncountable sets. Give examples for each of them.

(b) Prove that :

(i) Subset of a countable set is countable.

(ii) Superset of an uncountable set is un countable.

37. State and prove the Cauchy convergence criteria for a sequence.

38. (a) Define closed sets in R. Show that the Intersection of an arbitrary collection of closed sets in R is closed.

(b) Show by an example that the union of infinitely many closed sets in R need not be closed.

(1 × 15 = 15 marks)

D 92458-A (Pages : 4) Name.....................................

Reg. No.................................

FIFTH SEMESTER U.G. (CUCBCSS—UG) DEGREE [SPECIAL] EXAMINATION, NOVEMBER 2020

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 06—ABSTRACT ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. Which of the following defines a binary operation on Z^+ ?

   (A) $a * b = a - b$.

   (B) $a * b = c$, where c is the smallest integer greater than both a and b.

   (C) $a * b = c$, where c is at least 5 more than $a + b$.

   (D) $a * b = c$, where c is the largest integer less than the product of a and b.

2. If b and c are the inverses of some element a in a group G then :

   (A) $b = c$.

   (B) $b \neq c$.

   (C) b = kc for some $k \in N$.

   (D) None of these.

3. On Q, which of the following does not define a binary operation ?

   (A) $a * b = | a | b$.

   (B) $a * b = (a - b)^2$.

   (C) $a * b = +\sqrt{ab}$.

   (D) None of these.

4. Let * be the binary operation defined on Q^+ as $a * b\ ab/2$. Then inverse of the element a is :

   (A) $2a$.

   (B) $4/a$.

   (C) a^2.

   (D) None of these.

5. Which of the following are true ?

   (1) A group may have more than one identity element.

   (2) Any two groups of three elements are isomorphic.

   (3) Every group of at most three elements is abelian.

   (A) 2 and 3.

   (B) 1 and 2.

   (C) 1 and 3.

   (D) All.

6. Let G = $\{1, -1, i, -i\}$ where, be a set of four elements. Which of the following is a binary operation on G ? (1) $a * b = a + b$. (2) $a * b - a.b$.

   (A) Only 1.

   (B) Only 2.

   (C) Both.

   (D) None of these.

7. In a group G, $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$. This statement is :

   (A) Always true.

   (B) True if G is finite.

   (C) True if G is a multiplicative group.

   (D) True if G is abelian.

8. If a group G is of order 31, then which of the following is false ?

   (A) G is abelian.

   (B) G is cyclic.

   (C) G is abelian but not cyclic.

   (D) Both abelian and cyclic.

9. The Klein 4-group is isomorphic to ————.

   (A) $Z_2 \times Z_4$.

   (B) $Z_2 \times Z_2$.

   (C) Z_4.

   (D) None of these.

10. Order of (2, 2) in $Z_4 \times Z_6$ is ————.

    (A) 2

    (B) 6.

    (C) 4.

    (D) 12.

11. Which of the following is true ?

    (A) Every cyclic group has a unique generator.

    (B) In a cyclic group, every element is a generator.

    (C) Every cyclic group has at least two generators.

    (D) None of these.

12. Which of the following is a cyclic group with only one generator ?

    (A) Z_2.

    (B) (Z, +).

    (C) Klein-4 group.

    (D) None of these.

13. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$. Then σ^6 equals :

    (A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$

    (B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$.

    (C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$

    (D) None of these.

14. The product (1 3 6) (2 4) of two permutation is :

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 6 & 2 & 5 & 1 \end{pmatrix}$.

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$.

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 6 & 1 & 4 \end{pmatrix}$.

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 4 & 6 & 1 \end{pmatrix}$.

15. Which of the following is true ?

(A) Every function is a permutation if and only if it is one to one.

(B) The symmetric group S3 is cyclic.

(C) The symmetric group Sn is not cyclic for any n.

(D) Every function from a finite set onto itself must be one to one.

16. Which of the following is an even permutation ?

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 1 & 6 & 2 & 1 & 8 \end{pmatrix}$.

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 4 & 5 & 3 & 7 & 8 & 6 \end{pmatrix}$.

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 4 & 3 & 5 & 2 & 6 & 8 & 7 \end{pmatrix}$.

(D) None of these.

17. What is the largest possible order of a cyclic subgroup of $Z_{12} \times Z_{15}$?

(A) 60.

(B) 30.

(C) 180.

(D) None of these.

18. In a non-abelian group the element a has order 108. Then the order of a 12 is :

(A) 54.

(B) 27.

(C) 18.

(D) 9.

19. f is a homomorphism f :(R, +) $\rightarrow$ (Z, x) such that f(2) = 3. Then f(6) I is :

(A) 6.

(B) 9.

(C) 18.

(D) 27.

20. Which of the following is not true ?

(A) Every subgroup of every group has left cosets.

(B) A subgroup of a group is a left coset of itself.

(C) An is of index 2 in Sn for $n > 1$.

(D) None of these.

D 92458 (Pages : 4) Name.................................

Reg. No..............................

FIFTH SEMESTER U.G. DEGREE (SPECIAL) EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Fill in the blanks : Example of a finite abelian group of order 4 which is not cyclic is ————.
2. Fill in the blanks : Number of units in the ring $\mathbb{Z}_9$ is ————.
3. Fill in the blanks : One way of writing, to show that the identity permutation i is a transposition, is i = ————.
4. Give an example of a group of least order which is not abelian
5. Give an example of a finite integral domain.
6. Compute the inverse of the permutation $\sigma = (1)(1\ 2)(1\ 3)$ in S_4.
7. Show that the identity element in a group is unique.
8. Find all generators of the group $\mathbb{Z}_5$.
9. Define ring homomorphism.
10. Define permutation.
11. Define division ring.
12. Define a function from $n\mathbb{Z}$ to $\mathbb{Z}$ which is actually an isomorphism.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Show that every cyclic group is abelian.

14. Draw the subgroup diagram for $\mathbb{Z}_{12}$.

15. Show that a group is a finite group if it has finite number of subgroups.

16. Let S be a set and let ϕ, ψ and μ be functions mapping S into S. Prove that $\phi * (\psi * \eta) = (\phi * \psi) * \eta$ where the binary operation * is the function composition.

17. Show that intersection of arbitrary number of subgroups is again a subgroup.

18. Compute the product $\sigma\mu$ when $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 6 & 3 \end{pmatrix}$.

19. Show that any infinite cyclic group is isomorphic to $\mathbb{Z}$.

20. Prove that in any group G, $(a * b)^{-1} = b^{-1} * a^{-1}$ for all elements $a, b \in$ G.

21. Show that order of an element in a finite group divides the order of the group.

22. Show that the coset multiplication given by $(Ha)(Hb) = Hab$ is a well defined operation when H is a normal subgroup of G.

23. Find all left cosets of $H = \{0,3\}$ in $\mathbb{Z}_6$.

24. Calculate ϕ (4) if $\phi : \mathbb{Z} \rightarrow \mathbb{Z}$ is a homomorphism that satisfies ϕ (1) =1.

25. Is Q, the set of rationals, the field of quotients for integers ? Justify your claim.

26. Define binary operation and give an example of it which is not associative and also not commutative.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Show that every permutation σ of a finite set is a product of disjoint cycles.

28. Let G be a group. Show that the permutations, $\rho_a : G \to G$ defined by $\rho_a\ (x) = ax$, do form a group isomorphic to G.

29. Give any necessary and sufficient condition for a ring R to have no zero divisors, Justify your claim.

30. Define Kernal of a group homomorphism and show that it is a normal subgroup of the domain of the homomorphism.

31. Prove that every field L containing an integral domain D contains the field of quotients of D.

32. Find the index of the subgroup $\langle \sigma \rangle$ in S_5 when $\sigma = (1, 2, 5, 4)\ (2, 3)$.

33. If the index of a subgroup is 2, show that it is a normal subgroup.

34. If $H \leq G$ and if we define a relation R in G by $a R b$ if $a^{-1}\ b \in H$, show that it is an equivalence relation ?

35. Prove or disprove : (i) Every finite field is an integral domain ; and (ii) Every integral domain is a field.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) State and prove Lagranges theorem.

    (b) Express $\sigma = (1\ 2\ 3)^{-1}\ (1\ 3\ 4)^2 \in S_4$ as a product of disjoint cycles.

37. (a) If $\phi : G \rightarrow G'$ is a group homomorphism then show that image of a subgroup of G is a subgroup of G'.

(b) Show that the set of all even permutations form a subgroup of group of Permutations.

38. (a) Give an example of a finite group of order 3 and verify Cayley's theorem for that group.

(b) Find all the units in the ring $< Z_{12}, +_{12}, \times_{12} >$.

(1 × 15 = 15 marks)

D 92457-A **(Pages : 4)** **Name.....................................**

Reg. No...................................

FIFTH SEMESTER U.G. DEGREE [SPECIAL] EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 05—VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

1. The angle between the vectors $a = [1, 2, 3]$ and $b = [0, -2, 1]$ is :

(A) $\cos^{-1} \frac{1}{\sqrt{60}}$.

(B) $\cos^{-1} \frac{-1}{\sqrt{70}}$.

(C) $\cos^{-1} \frac{1}{\sqrt{70}}$.

(D) $\cos^{-1} \frac{1}{\sqrt{80}}$.

2. The straight line through the point $(1, 3)$ in the xy plane and perpendicular to the straight line $x - 2y + 2 = 0$ is :

(A) $3x - y = 2$.

(B) $x + y = 1$.

(C) $2x + y = 5$.

(D) $2x - y = 5$.

3. The parametric equations for the line through $(-3, 2, -3)$ and $(1, -1, 4)$ are :

(A) $x = 1 + 4t,\ y = -1 - 3t,\ z = 4 + 7t$.

(B) $x = 2 + 4t,\ y = -2 - 3t,\ z = -4 + 7t$.

(C) $x = 3 + 4t,\ y = 8 - 3t,\ z = 5 + 7t$.

(D) $x = 1 - 4t,\ y = -1 + 3t,\ z = -4 - 7t$.

4. The point of intersection of the line $x = \frac{8}{3} + 2t,\ y = -2t,\ z = 1 + t$ and the plane $3x + 2y + 6z = 6$ is :

(A) $(1, 1, 2)$.

(B) $(2, 0, 1)$.

(C) $\left(\frac{2}{3}, 2, 0\right)$.

(D) $(0, 1, 3)$.

5. If $r(t) = \sin ti + e^{-t} y + 3k$, then $\frac{dr}{dt}$ is :

(A) $\sin ti + 3k$.

(B) $\cos ti + e^{-t} j + 3k$.

(C) $\cos ti - e^{-t} j$.

(D) $\sin ti - e^{-t} j$.

6. The domain of the function $f(x, y, z) = xy \ln(z)$:

(A) Entire Space

(B) $\{(x, y, z) : xyz \neq 0\}$.

(C) Half space $z > 0$.

(D) Hald space $z < 0$.

7. Which of the following holds for the function $f(x, y) = \frac{4x^6 y^2}{x^{12} + y^4}$?

(A) $\lim_{(x, y) \to (0, 0)} f(x, y)$ exists.

(B) $\lim_{(x, y) \to (0, 0)} f(x, y)$ doesn't exists.

(C) $\lim_{(x, y) \to (0, 0)} f(x, y) = 0$.

(D) None of these.

8. Let $f(x, y) = x - y$ and $g(z, y) = e^z$ be two continuous functions. Then the composition function $g(f(x, y)) = e^{x-y}$ is :

(A) Discontinuous.
(B) Continuous.
(C) Continuous at origin.
(D) None of these.

9. The function $f(x, y) = xy$ has a :

(A) Local maximum.
(B) Local minimum.
(C) Both local maximum and minimum.
(D) No local extreme values.

10. The minimum value that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ is :

(A) 2.
(B) – 2.
(C) 4.
(D) – 4.

11. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. The points on the ellipse that lies closest to the origin are :

(A) $(1, 0, 0)$ and $(0, 0, 1)$.
(B) $(0, 1, 0)$ and $(0, 0, 1)$.
(C) $(1, 0, 0)$ and $(0, 1, 0)$.
(D) $(1, 0, 0)$ and $(0, 1, 1)$.

12. What is the value of $\iint xy\,dx\,dy$ over the first guadrant of the circle $x^2 + y^2 = a^2$?

(A) $\frac{a^2}{4}$.
(B) $\frac{a^2}{8}$.
(C) $\frac{a^4}{4}$.
(D) $\frac{a^4}{8}$.

13. A coil spring lies along the helix $r(t) = (\cos 4t)\,\mathrm{I} + (\sin 4t)\,j + k, 0 \le t \le 2\pi$. The spring's densi... constant, $\delta = 1$. Then the radius of gyration of the spring about the z-axis is :

(A) 1.
(B) 2.
(C) 3.
(D) 4.

14. The gradient field of $f(x, y, z) - xyz$ is :

(A) $yzi + xzj + xyk$.
(B) $xyi + xzj + yzk$.
(C) $xzi + yzj + xyk$.
(D) None of these.

15. If $\nabla\varnothing = \left(y + y^2 + z^2\right) i + \left(x + z + 2xy\right) j + \left(y + 2xz\right) k$ and $\varnothing\left(1,1,1\right) = 3$, then what is $\varnothing$?

(A) $xz + xy + yz^2 - 1$.
(B) $xz + yz + xz^2$.
(C) $xy^2 + xz^2 - 1$.
(D) $xy + xy^2 + xz^2 + yz - 1$.

16. Which among the following is the work done in moving a particle once round a circle C in the xy-plane. Given the circle has centre at the origin and radius 3 and the force field is given by $F = \left(2x - y + z\right) i + \left(x + y - z^2\right) j + \left(3x - 2y + 4z\right) k$.

(A) 8π.
(B) 80π.
(C) 88π.
(D) 18π.

17. If $F = \left(3x^2 + 6y\right) j - 14yzj + 20xz^2k$, then the value of $\int_C F \cdot dr$ where C is a curve from $\left(0,0,0\right)$ to $\left(1,1,1\right)$ with parametric from $x = t, y = t^2, z = t^3$ is :

(A) 13.
(B) 7.
(C) 5.
(D) 11.

18. If $r^\wedge$ is the unit vector in the direction of r and $r = \left| r \right|$, then div $\left(r^\wedge\right)$ is :

(A) $\frac{r}{2}$.
(B) r.
(C) zr.
(D) $\frac{z}{r}$.

19. Vector product is :

(A) Commutative.
(B) Anticommutative.
(C) Associative.
(D) Not distributive wet vector addition.

20. If F,G are differentiable vector functions and $\varnothing$ is a differentiable scalar function. Then :

(A) curl $\left(F \times G\right) = \left(\text{grad } \varnothing\right) \times F + \varnothing$ curl $\left(F\right)$.

(B) div $\left(F \times G\right) = -$ F curl G + g curl F.

(C) div $\left(F \times G\right) = \left(G, \nabla\right) F - \left(F \nabla\right) G + F$ div G $-$ G div F.

(D) curl $\left(F \times G\right) = F$ curl G $-$ G curl F.

D 92457 **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER U.G. DEGREE [SPECIAL] EXAMINATION NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the domain and range of $f(x, y) = \cos xy$.

2. Evaluate $\lim\limits_{(x, y)\to(0, 0)} \dfrac{y}{x}$.

3. Define gradient of $\phi(x, y, z) = xyz$.

4. Compute the divergence of $\vec{f} = y^2\,\vec{i} + z^2\,\vec{j} + x^2\,\vec{k}$.

5. What do you mean by the directional derivative of a function ?

6. When do you say that a vector field is conservative ?

7. What is the linearization of the function $f(x, y, z)$ at the point (x_0, y_0, z_0) ?

8. Find the total differential of u if $u = \ln\left(x^2 + y^2 + z^2\right)$.

9. What do you mean by circulation around a curve ?

10. State the Normal form of Green's theorem in the plane.

11. Fill in the blanks : If $\vec{a}$ is a constant vector and $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, then $\nabla \times \left(\vec{a} \times \vec{r}\right) =$ ————————.

12. State Stokes theorem.

(12 × 1 = 12 marks)

Turn over

Section B

Answer at least **eight** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 48.

13. Evaluate $\lim_{(x, y) \to (0, 0)} \dfrac{x - y}{y + x}$.

14. Find the vector normal to the surface $\phi(x, y, z) = x^2 + y^2 + z^2$ at $(1, 1, 1)$.

15. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(1, 1, 1)$, if $x^2 + y^2 + z^2 + ye^x z + z \cos y = 0$.

16. Prove that $\nabla \cdot \left(r^n \vec{r}\right) = (n + 3) \vec{r}$.

17. Find the total derivative of $u = x^3 + y^3$ with respect to t if $x = a \cos t$, $y = b \sin t$.

18. Compute the average value of the function $f(x, y) = x \cos(xy)$ over the rectangular region $0 \le x \le \pi, 0 \le y \le 1$.

19. Find the directional derivative of $f(x, y) = xe^y + \cos(xy)$ at $(2, 0)$ in the direction of $3\vec{i} - 4\vec{j}$.

20. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix

    $x = \cos t, y = \sin t, z = t; \; 0 \le t \; \pi/2$.

21. Find the value of λ which makes the following vector

    $\vec{f} = \left(\lambda xy - z^3\right)\vec{i} + \left((\lambda - 2)x^2\right)\vec{j} + \left((1 - \lambda)xz^2\right)\vec{k}$ is irrotational.

22. Verify whether the differential $e^x \cos y \, dx + \left(xz - e^x \sin y\right) dy + (xy + z) \, dz$ is exact or not.

23. If $\vec{f}$ and $\vec{g}$ are irrotational, then show that $\vec{f} \times \vec{g}$ is solenoidal.

24. Evaluate $\int_0^1 \int_0^2 xy\,(x - y)\,dydx.$

25. Test whether the vector $\vec{f} = \left(e^x \cos y + yz\right)\vec{i} + \left(xz - e^x \sin y\right)\vec{j} + (xy + z)\vec{k}$ is conservative or not.

26. Prove *or* disprove : If div $\vec{f} = 0$, then curl $\vec{f} = 0$.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dydx.$

28. Find $\vec{\nabla} f(r)$, if $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}.$

29. Find the work done by the force field $\vec{f} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ along any of the path joining the points $(-1, 3, 9)$ and $(1, 6, -4)$.

30. Evaluate the outward flux of $\vec{f} = 4xz\,\vec{i} - y^2\,\vec{j} + yz\,\vec{k}$ through the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

31. Find the equation to the tangent plane and normal line to the surface $f(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point $(1, 2, 4)$.

32. Evaluate the area enclosed by one leaf of the rose $r = 12\cos 3\theta$.

33. Find the greatest and the smallest values of $f(x, y) = xy$ takes on the ellipse $x^2/8 + y^2/2 = 1$.

34. Evaluate $\int_{(0,0,0)}^{(1,2,3)} 2xydx + \left(x^2 - z^2\right) dy - 2yzdz.$

35. Show that $\vec{f} = (x + 2y + 4z)\vec{i} + (2z - 3y - z)\vec{j} + (4x - y + 2z)\vec{k}$ is conservative and find its scalar potential.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. Verify Gauss's divergence theorem for $\vec{f} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ over the sphere of radius a centred at the origin.

37. (a) Evaluate $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz.$

(b) If S is a closed surface enclosing a volume V, then prove that $\iint\limits_{S} (\text{curl})\,\vec{f} \cdot n dS = 0.$

38. Using Green's theorem find the counter clockwise circulation and outward flux for the field $\vec{f} = \left(x^2 + 4y\right)\vec{i} + \left(x + y^2\right)\vec{j}$ and the square C bounded by $x = 0, x = 1, y = 0, y = 1.$

(1 × 15 = 15 marks)

D 10674 **(Pages : 2)** Name.....................................

Reg. No.................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5D 04—MATHEMATICS FOR DECISION-MAKING

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define Statistics.
2. Distinguish between population and sample.
3. What is meant by frequency distribution ?
4. Define Median.
5. Write down the sample space when a six sided die is rolled.
6. Identify the type of probability in the statement : The probability that you will get an A on your next test is 0.9.
7. Define mutually exclusive events.
8. State the addition rule of probability of two events.
9. What is a discrete random variable ?
10. Find the missing probability from the following probability distribution :

x	:	0	1	2	3	4
P (x)	:	0.07	0.20	0.38	?	0.13

11. Find the mean and variance of a binomial distribution with $n = 50$ and $p = 0.4$.
12. In a geometric distribution, what does the random variable measure ?

(8 × 3 = 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Explain the different levels of measurement of data.

14. What is the difference between stratified sample and systematic sample ?

15. Give the Classical and Empirical definitions of probability.

16. A coin is tossed and then a die is rolled. Find the probability of tossing a head and then rolling a 6.

17. A corporation has six male senior executives and four female senior executives. Four senior executives are chosen at random to attend a technology seminar. What is the probability of choosing two male and two female senior executives ?

18. The number of hours students in a college class slept the previous night is given below :

Hour	:	4	5	6	7	8	9	10
Students		1	6	8	20	9	4	2

Construct a probability distribution.

19. A particular surgery has 90 % chance of success. The surgery is performed on three patie nts. Find the probability of the surgery being successful on exactly two patients.

(5 × 5 = 25 marks

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. The marks of students in two subjects A and B are given below. Find the co-efficie it of variation and compare the results.

A		35	70	45	62	55	72	90	44	62	75
B	:	88	73	55	82	48	95	82	95	48	84

21. About 60 % cancer survivors are ages 65 years and older. Six cancer survivors are selected at random ask them whether they are 65 years of age and older. Construct a probability distribution for the number of cancer survivors of this age group. Also find the mean, variance and standard deviation.

(1 × 11 = 11 marks)

D 10673-A **(Pages : 4)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5D 03—LINEAR MATHEMATICAL MODELS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 15**	**Maximum : 15 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5D 03—LINEAR MATHEMATICAL MODELS

(Multiple Choice Questions for SDE Candidates)

1. What is the slope of line passing through the points (0,1) and (1,1) ?

   (A) 1. (B) –1.

   (C) 0. (D) 0.5.

2. Equation of a line whose x-intercept is 1 and y-intercept is –1.

   (A) $x + y = 1$. (B) $x - y = 1$.

   (C) $x + y = -1$. (D) $x - y = -1$.

3. P : Slope of a horizontal line is 1.

   Q : Slope of a vertical line is undefined.

   (A) P is true and Q is false. (B) P is false and Q is true.

   (C) P and Q are false. (D) P and Q are true.

4. Find the equation of the line that passes through the point (3, 5) and is parallel to the line $2x + 5y = 4$.

   (A) $5x + 2y = 25$. (B) $2x - 5y = -19$.

   (C) $2x + 5y = 31$. (D) $2x + 5y = -19$.

5. Find the equation of the line that passes through the point (0,1) and is perpendicular to the line $x = 0$.

   (A) $y = 0$. (B) $y = 1$.

   (C) $y = -1$. (D) None of the above.

6. What is the solution of the system of linear equation given below ?

   $x + y = 1$
   $x - y = -1$

   (A) $(0, 1)$. (B) $(0, -1)$.

   (C) $(1, 0)$. (D) $(-1, 0)$.

7. $\begin{matrix} x - 3y = 1 \\ x - 3y = -1 \end{matrix}$ the linear system of equation has :

(A) Unique solution. (B) Infinitely many solutions.

(C) No solution. (D) None of the above.

8. $\begin{matrix} 2x - 6y = 4 \\ 3x - 9y = 6 \end{matrix}$ the linear system of equation has :

(A) Unique solution. (B) Infinitely many solutions.

(C) No solution. (D) None of the above.

9. $\begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix} =$ ————————.

(A) $\begin{bmatrix} -1 & 10 \\ 16 & 6 \end{bmatrix}$. (B) $\begin{bmatrix} 1 & 0 \\ 16 & 6 \end{bmatrix}$.

(C) $\begin{bmatrix} -1 & 12 \\ 16 & -6 \end{bmatrix}$. (D) $\begin{bmatrix} -1 & 0 \\ 16 & -6 \end{bmatrix}$.

10. Let A be a matrix of order $m \times n$ and B be a matrix of order $p \times q$. Then A B is defined if and only if :

(A) $m = p$. (B) $m = q$.

(C) $n = p$. (D) $n = q$.

11. $A = \begin{bmatrix} 5 & -6 \\ 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 6 \\ 8 & -3 \end{bmatrix}$. Then AB = ————————.

(A) $\begin{bmatrix} -68 & 48 \\ 40 & 19 \end{bmatrix}$. (B) $\begin{bmatrix} -68 & 48 \\ -104 & 21 \end{bmatrix}$.

(C) $\begin{bmatrix} -68 & 42 \\ 40 & 21 \end{bmatrix}$. (D) $\begin{bmatrix} -68 & 48 \\ 40 & 21 \end{bmatrix}$.

12. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ then $A^{-1} =$ ————.

(A) $\begin{bmatrix} -5 & 3 \\ -2 & -1 \end{bmatrix}$.

(B) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$.

(C) $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$

(D) None of the above.

13. Let A be a square matrix.

P : A^{-1} exist for every square matrix A. Q : There are square matrices A whose inverse does not exist.

(A) P is true and Q is false.

(B) P is false and Q is true.

(C) P and Q are false.

(D) P and Q are true.

14. The maximum value of the objective function $z = 3x + 4y$, subject to the following constraints

$$2x + y \le 4$$
$$-x + 2y \le 4$$
$$x \ge 0$$
$$y \ge 0$$

(A) 6.

(B) 12.

(C) 18.

(D) 24.

15. What is the solution of the following linear programming problem ?

Minimize $Z = 2x + 4y$

subject to $x + 2y \ge 10$
$3x + y \ge 10$
$x \ge 0$
$y \ge 0.$

(A) 5.

(B) 10.

(C) 15.

(D) 20.

D 10673 (Pages : 3) Name..

Reg. No.....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS–UG)

Mathematics

MTS 5D 03—LINEAR MATHEMATICAL MODELS

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Does the line $y = -x + 5$ intersect the point $(3, -1)$? Why ?

2. Let $g(x) = -4x + k$ where k is a constant. If $g(3) = 5$, find the value of k.

3. Solve the system of equations $3x + y = 5$, $3x = 6$.

4. Write the augmented matrix for the system of equations $3x + y = 6$, $2x + 5y = 15$.

5. Find values of x, y if $\begin{bmatrix} 3 & 4 \\ -8 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2y \\ x & 1 \end{bmatrix}$.

6. Find the product of matrices : $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -1 & 5 \\ 0 & 3 \end{bmatrix}$.

7. Graph the linear inequality $x \le 3y$.

8. Define the term *corner point*. State the corner point theorem.

9. Give an example for a maximization problem in standard form with 2 variables.

10. Sketch the feasible region for the linear programming problem :

    Maximize $Z = 2x + 3y$ subject to $x \ge 0, y \ge 0$.

11. What are the conditions to be satisfied to call a linear programming problem to be in standard minimum form ?

Turn over

12. Write the matrix form of the linear programming problem :

Minimize $W = 8y_1 + 16y_2$

subject to
$$y_1 + 5y_2 \geq 9$$
$$2y_1 + 2y_2 \geq 10$$
$$y_1 \geq 0, y_2 \geq 0.$$

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. In recent years, the percentage of the U.S. population age 18 and older who smoke has decreased at a roughly constant rate, from 24.1% in 1998 to 20.6% in 2008. Find the equation describing this linear relationship.

14. Solve the system of equations :

$$3x + 10y = 115$$
$$11x + 4y = 95$$

using echelon method.

15. A convenience store sells 23 sodas one summer afternoon in 12-, 16-, and 20-*oz* cups (small, medium, and large). The total volume of soda sold was 376 *oz*. Suppose that the prices for a small, medium, and large soda are $1, $1.25, and $1.40, respectively, and that the total sales were $28.45. How many of each size did the store sell ?

16. Solve the following system of equations using the inverse of the coefficient matrix :

$$x + 3y - 2z = 4$$
$$2x + 7y + 3z = 8$$
$$3x + 8y + 5x = -4.$$

17. Graph the feasible region for the following system of inequalities and tell if it is bounded or unbounded :

$$3x - 2y \geq 6$$
$$x + y \leq -5$$
$$y \leq -6$$

18. Find the maximum value of the objective function $z = 3x + 4y$, subject to the constraints : $-x + 2y \leq 4, x \geq 0, y \geq 0.$

19. Add slack variables to the following linear programming problem and write the initial simplex tableau:

Maximize $Z = 3x_1 + 2x_2 + x_3$

subject to $2x_1 + x_2 + x_3 \leq 150$

$2x_1 + 2x_2 + 8x_3 \leq 200$

$2x_1 + 3x_2 + x_3 \leq 320$

and $x_1, x_2, x_3 \geq 0.$

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Assume that the demand of an item A increases as the price decreases. If the weekly demand for A is q (in dollar \$) and the price is p then suppose that they are related by the linear relation $D(p) = 9 - 0.75q$.

(a) Find the quantity demanded at a price of \$ 5.25 per item and at a price of \$ 3.75 per item.

(b) It is also noticed that the quantity of item A supplied decreased as the price decreased. If price p and supply q are related by the linear function $S(p) = 0.75q$, find the quantity supplied at a price of \$ 5.25 per item and at a price of \$ 3.00 per item.

(c) Graph both functions $D(p)$ and $S(p)$ on the same axes.

21. Solve the problem using simplex method :

Maximize $Z = 12x_1 + 15x_2 + 5x_3$

subject to $2x_1 + 2x_2 + x_3 \leq 8$

$x_1 + 4x_2 + 3x_3 \leq 12$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

(1 × 11 = 11 marks)

D 10672 (Pages : 2) Name...

Reg. No.....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5D 02—DISCRETE MATHEMATICS FOR BASIC AND APPLIED SCIENCES

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define a proposition.
2. What is disjunction of two propositions ? Explain using its truth table.
3. Show that $p \wedge \overline{p}$ is a contradiction.
4. When do we say that two propositions are logically equivalent ?
5. Define binary operation.
6. Give example of an abelian monoid.
7. Give example of a cyclic group.
8. Give example of a semi group.
9. Define regular graph.
10. Define a complete bipartite graph.
11. When do we say that a graph is connected ?
12. Define a Hamiltonian graph.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Construct truth table for the compound proposition $(p \to q) \wedge (q \to p)$.
14. Show that $\overline{(p \wedge q)}$ and $\overline{p} \vee \overline{q}$ are logically equivalent.

Turn over

15. For an associative binary operation on a set S which has identity element, show that inverse if it exists is unique.

16. Show that the group $(\mathbb{Z},+)$ is cyclic with generator 1 ?

17. For a Boolean algebra state and prove De Morgan's laws ?

18. Draw diagrams to represent the complete graph K_2 and the complete bipartite graph $K_{2,5}$.

19. Draw the diagram of sequence (2, 2, 2, 4, 4, 5, 5).

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Describe the degree sequence of :

    (a) The null graph with n vertices.

    (b) The complete graph K_n.

    (c) An r-regular graph with n vertices.

    (d) The complete bipartite graph $K_{n,m}$ where $n \leq m$.

21. Construct truth tables for :

    (a) $\neg(p \to q) \to \neg p$.

    (b) $q \leftrightarrow (\neg p \vee \neg q)$.

(1 × 11 = 11 marks)

D 10671 **(Pages : 3)** **Name...**

Reg. No.....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5D 01—APPLIED CALCULUS

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. If $g(t) = (t-2)^{\frac{1}{2}}$, find $g(27)$ and $g(5)$?

2. Find the domain and range of $f(x) = \dfrac{1}{x-3}$.

3. Find $f(g(x))$ where $f(u) = u^2 + 3u + 1$ and $g(x) = x + 1$

4. Find the distance between the points $P(-2, 5)$ and $Q(4, -1)$.

5. State vertical line test.

6. What is slope of (a) X-axis ; and (b) Y- axis.

7. Find the equation of the line that passes through the points $(3, -2)$ and $(1, 6)$.

8. Find $\lim\limits_{x \to 1} \dfrac{x^3 - 8}{x - 2}$.

9. Differentiate the polynomial $y = 5x^3 - 4x^2 + 12x - 8$.

Turn over

10. Find all critical numbers of the function $f(x) = 2x^4 - 4x^2 + 3$.

11. Find all real numbers x that satisfy the given equation $4^{2x-1} = 16$.

12. Evaluate $\log_2 32$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find :

(a) $\lim\limits_{x \to \infty} \dfrac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$.

(b) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - 3x + 2}$.

14. For the function $f(x) = 1 - x^2, 0 \le x < 2$
$= 2x + 1, x \ge 2$.

evaluate the one sided limits $\lim\limits_{x \to 2-} f(x)$ and $\lim\limits_{x \to 2+} f(x)$.

15. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 4$.

16. Find the intervals of increase and decrease for the function $x^2 - 4x + 5$.

17. If $f(x) = 5^{x^2 + 2x}$ find all values of x such that $f(x) = 125$.

18. Discuss the continuity of the function $f(x)=\frac{x+2}{x-3}$ on $-2<x<3$ and on the closed interval $-2\leq x\leq 3$.

19. The Gross Domestic Product (GDP) of a certain country was $N(t)=t^2+5t+106$ billion dollars t years after 1998 :

    (a) At what rate was the GDP changing with respect to time in 2008.

    (b) At what percentage rate was the GDP changing with respect to time in 2008.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Graph the function $f(x)=-x^2+x+2$. Include all x and y intercepts.

21. A manufacturer's total cost consists of a fixed overhead of \$200 plus production cost of \$50 per unit. Explore the total cost as a function of the number of units produced and draw graph.

(1 × 11 = 11 marks)

D 10670-A (Pages : 4) Name.....................................

Reg. No.....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5B 09—INTRODUCTION TO GEOMETRY

(Multiple Choice Questions for SDE Candidates)

1. Find the equation of the circle with center (– 2, 3) and radius 4 :

   (A) $x^2+y^2+4x-6y-3=0.$
   (B) $x^2+y^2+-3=0.$
   (C) $x^2+y^2+x-6y-1=0.$
   (D) $x^2+y^2+-6y-3=0.$

2. Find the center and radius of the circle $(x+5)^2+(y-3)^2=36$:

   (A) Center (5, 3) and radius 6.
   (B) Center (– 5,3) and radius 6.
   (C) Center (0,0) and radius 7.
   (D) Center (– 5,2) and radius 5.

3. Does the points (–2.5, 3.5) lie inside, outside or on Phe circle $x^2+y^2=25$:

   (A) (–2.5, 3.5) lies outside the circle.
   (B) (–2.5, 3.5) lies inside the circle.
   (C) (–2.5, 3.5) lies on the circle.
   (D) None of these.

4. Find the length of the latus rectum for $y^2=12x$:

   (A) 3.
   (B) – 3.
   (C) 12.
   (D) – 12.

5. Find the length of the latus rectum for $y^2=-8x$:

   (A) – 8.
   (B) 8.
   (C) 12.
   (D) – 12.

6. Find the axis of the parabola $x^2=-16y$:

   (A) x-axis.
   (B) y-axis.
   (C) z-axis.
   (D) None of this.

7. Find the equation of directrix of the parabola $y^2=10x$:

   (A) $y+\frac{5}{2}=0.$
   (B) $x+\frac{5}{2}=0.$
   (C) $x-5=0.$
   (D) $y+5=0.$

8. Find the equation of the parabola that satisfies the following conditions: focus (0, 0), directrix $y = 3$.

(A) $x^2 = -12y$.
(B) $y^2 = 25x$.
(C) $x^2 = 25y$.
(D) $y^2 = 12x$.

9. Find the co-ordinates of she focii of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

(A) $(\pm 2\sqrt{5}, 0)$.
(B) $(\pm \sqrt{5}, 0)$.
(C) $(\pm 10, 0)$.
(D) $(\pm \sqrt{10}, 0)$.

10. Find the co-ordinates of the vertices of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

(A) $(\pm \sqrt{5}, 0)$.
(B) $(0, \pm 5)$.
(C) $(\pm 5, 0)$.
(D) $(\pm \sqrt{5}, 0)$.

11. Find the co-ordinates of the focii of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(A) $(0, \pm \sqrt{7})$.
(B) $(\pm \sqrt{7}, 0)$.
(C) $(0, \pm\sqrt{25})$.
(D) $(\pm\sqrt{25}, 0)$.

12. The tangent to an ellipse and making angles 60° with x-axis is ?

(A) $\sqrt{3}x - y + 7 = 0$.
(B) $x + y + 3 = 0$.
(C) $x - 2y = 0$.
(D) $2x + y - 4 = 0$.

13. The equation of the tangent to the parabola with parametric equations $x = 2t^2$, $y = 4t$ at the point with parameter $t = 3$ is :

(A) $2y = x + 6$.

(B) $y = x + 6$.

(C) $3y = x + 18$.

(D) $y = x + 18$.

14. Identify the curve from the equation $7x^2 - 9y^2 = 36$.

(A) A parabola.

(B) An ellipse.

(C) A cycle.

(D) A hyperbola.

15. The matrix of the non-degenerate conic $x^2 + 8xy + 16y^2 - x + 8y - 12 = 0$:

(A) $\begin{bmatrix} 1 & 4 \\ 4 & 16 \end{bmatrix}$.

(B) $\begin{bmatrix} 3 & -4 \\ -4 & 2 \end{bmatrix}$.

(C) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

D 10670 (Pages : 3) Name..

Reg. No....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Find the equation of the tangent at the point with parameter t to the parabola with parametric equations $x = at^2, y = 2at$ where $t \in \mathbb{R}$.

2. Let E be a parabola with parametric equations $x = t^2, y = t, t \in \mathbb{R}$. Find focus, vertex axis and directrix of E.

3. Prove that the equation of the tangent at the point (x_1, y_1) to an ellipse is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

4. Write the equation of the conic $x^2 - 4xy + 4y^2 - 6x - 8y + 5 = 0$ in matrix form.

5. Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal for each real number θ.

6. Let the Euclidean transformations t_1 and t_2 of $\mathbb{R}^2$ be given by :

$$t_1(X) = \begin{bmatrix} \frac{3}{5} & \frac{-4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} X + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \text{ and}$$

$$t_2(X) = \begin{bmatrix} \frac{-4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} x + \begin{bmatrix} -2 \\ 1 \end{bmatrix}. \text{ Find } t_2 \circ t_1.$$

Turn

7. Find the inverse of the affine transformation $t(X) = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} X + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

8. State fundamental theorem of affine geometry.

9. Prove that an affine transformation maps straight lines to straight lines.

10. State Desargue's theorem.

11. Find the equation of the line that passes through the point $[1, 2, 3]$ and $[2, -1, 4]$.

12. Find the point of intersection of the lines in $\mathbb{RP}^2$ with equations $x + 6y - 5z = 0$ and $x - 2y + z = 0$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Let PQ be an arbitrary chord of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let M be the midpoint of PQ. Prove that the following expression is independent of the choice of P and Q : Slope of OM × Slope of PQ.

14. State and prove reflection properties of the ellipse.

15. Prove that the set of all affine transformations A(2) forms a group under the operation of composition of functions.

16. Determine the image of the line $y = 2x$ under the affine transformation $t(X) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} X + \begin{pmatrix} 2 \\ -1 \end{pmatrix}, X \in \mathbb{R}^2$.

17. Determine the affine transformation which maps the points $(2, 3), (1, 6)$ and $(3, -1)$ to the points $(1, -2), (2, 1)$ and $(-3, 5)$ respectively.

18. Prove that affine transformations maps ellipses to ellipses, parabolas to parabolas and hyperbolas to hyperbolas.

19. Determine the point of $\mathbb{RP}^2$ at which the line through the points $[1, 2, -3]$ and $[2, -1, 0]$ meets the line through the points $[1, 0, -1]$ and $[1, 1, 1]$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Prove that the conic E with equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola. Determine its centre, and its major and minor axes.

21. State and prove Ceva's theorem.

(1 × 11 = 11 marks)

D 10669-A **(Pages : 4)** **Name..................................**

Reg. No..................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 08—LINEAR PROGRAMMING

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 15**	**Maximum : 15 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5B 08—LINEAR PROGRAMMING

(Multiple Choice Questions for SDE Candidates)

1. "Any unbounded linear programming problem has an unbounded constraint set" :

   (A) The above statement is always TRUE.

   (B) The above statement is always FALSE.

   (C) The above statement can be TRUE or FALSE.

   (D) Insufficient data.

2. A linear programming problem having 6 main constraints and 3 non-negativity constraints. Find the upper bound for the number of extreme point candidates.

   (A) 504. (B) 120.

   (C) 20. (D) 84.

3. Which of the following is/are non-convex set(s) ?

   (i) x-axis, (ii) $R^2 - \{(0,0)\}$, (iii) Unit Circle.

   (A) All the above. (B) (i), (iii) but not (ii).

   (C) (ii) and (iii) but not (i). (D) Only (i).

4. The variables to the south of the minimum tableau are called :

   (A) Basic variables. (B) Slack variables.

   (C) Non-basic variables. (D) Independent variables.

5. A pivot entry is :

   (A) Always negative. (B) Always positive.

   (C) Always zero. (D) Always non-zero.

6. In non-canonical maximum tableaus :

   (A) Rows corresponding to pivoted unconstrained variables are filed and deleted.

   (B) Rows corresponding to pivoted unconstrained variables are filed and never deleted.

   (C) Columns corresponding to pivoted unconstrained variables are filed and deleted.

   (D) Columns corresponding to pivoted unconstrained variables are filed and never deleted.

7. Slack "variables" of 0 (corresponding to equations of constraint) in non-canonical maximum tableaus always :

   (A) Get pivoted up-from east to north.

   (B) Get pivoted up-from north to east.

   (C) Get pivoted up-from west to south.

   (D) Get pivoted up-from south to west.

8. Unconstrained independent variables in non-canonical maximum tableaus always :

   (A) Get pivoted down-from north to east.

   (B) Get pivoted down-from south to west.

   (C) Get pivoted down-from west to south.

   (D) Get pivoted down-from east to north.

9. "In a minimum basic feasible tableau, the basic solution is a feasible solution" :

   (A) FALSE. (B) TRUE.

   (C) May be TRUE. (D) May be FALSE.

10. Given dual canonical linear programming problem if Maximization problem has optimal solution then Minimization problem has :

   (A) Unbounded solution.

   (B) Infeasible solution.

   (C) Unbounded solution or infeasible solution.

   (D) Optimal solution.

11. In L.P.P. unbounded solution means :

   (A) Infeasible solution. (B) Degenerate Solution.

   (C) Infinite solution. (D) Unique Solution.

12. Assignment problem is a special type of transportation problem :

   (A) Always true. (B) May be true.

   (C) Always false. (D) May be false.

13. To find initial feasible solution of a transportation problem, the method which starts allocation from the minimum cost is called ——————.

(A) Minimum entry method.

(B) Northwest corner method.

(C) Northeast corner method.

(D) VAM method.

14. If the number of rows and columns in an assignment problem are not equal then it is said to be :

(A) Bounded.

(B) Infeasible.

(C) Unbounded.

(D) Unbalanced.

15. If there are n workers and n jobs and each worker assign to only one job then there would be :

(A) n ! solutions.

(B) $(n - 1)$! solutions.

(C) $(n + 1)$! solutions.

(D) n solutions.

D 10669 (Pages : 4) Name.......................................

Reg. No..................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 08—LINEAR PROGRAMMING

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define canonical minimization linear programming problem.

2. Give an example of a bounded polyhedral convex subset in $\mathbb{R}^2$.

3. State the canonical minimization linear programming problem represented by the following tableau :

x	1	2	3
y	4	5	6
-1	7	8	9
	$= t_1$	$= t_2$	g

4. Define unbounded linear programming problem.

5. Pivot on 5 in the canonical maximum tableau given below :

x_1	x_2	-1	
1	2	3	$= -t_1$
4	5	6	$= -t_2$
7	8	9	$= f$

6. Write the simplex algorithm for maximum tableaus.

7. What do you mean by complementary slackness ?

8. State Duality theorem.

Turn over

9. Consider the canonical maximization linear programming problem given below ;

Maximize $f(x_1, x_2) = x_1$ subject to

$$x_1 + x_2 \leq 1$$
$$x_1 - x_2 \geq 1$$
$$x_2 - 2x_1 \geq 1$$
$$x_1, x_2 \geq 0$$

state the dual canonical minimization of the linear programming problem.

10. Distinguish between balanced and unbalanced transportation problem.

11. Using VAM to obtain a basic feasible solution of the transportation problem given below :

4	5	5
3	2	7
6	3	9
7	5	4
14	11	

12. Explain the minimum entry method for obtaining initial basic feasible solution in transportation problem.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Solve the following linear programming problem by geometrical method.

Maximize $f(x, y) = -2y - x$ subject to

$$2x - y \geq -1$$
$$3y - x \leq 8$$
$$x, y \geq 0.$$

14. Solve the following canonical linear programming problem using simplex algorithm :

x_1	x_2	-1	
−1	1	1	$= -t_1$
1	−1	3	$= -t_2$
1	2	0	$= f$

15. Solve the canonical linear programming problem using simplex algorithm :

x	– 2	1	– 3
y	1	– 2	– 2
– 1	1	0	0
	$= t_1$	$= t_2$	g

16. Solve the non-canonical linear programming problem given below

Maximize $f(x,y,z) = 2x + y - 2z$ subject to

$x + y + z \leq 1$
$y + 4z = 2$
$x, y, z \geq 0.$

17. Write the dual simplex algorithm for minimum tableaus.

18. Solve the transportation problem given below :

	M_1	M_2	M_3	
W_1	2	1	2	50
W_2	9	4	7	70
W_3	1	2	9	20
	40	50	20	

19. Apply Northwest-corner method to obtain the initial basic feasible solution of the transportation problem given below :

7	2	4	10
10	5	9	20
7	3	5	30
20	10	30	

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Solve the canonical linear programming problem given below using the simplex algorithm.

x	y	z	– 1	
1	2	1	4	$= -t_1$
2	1	5	5	$= -t_2$
3	2	0	6	$= -t_3$
1	2	3	0	$= f$

21. Write the Hungarian algorithm. Using this algorithm solve the following assignment problem :

2	3	2	4
5	8	4	3
5	9	5	2
7	6	7	4

(1 × 11 = 11 marks)

D 10668–A (Pages : 4) Name..

Reg. No..

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 15	Maximum : 15 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 15.
2. The candidate should check that the question paper supplied to him/her contains all the 15 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5B 07—NUMERICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. The floating-point form of π using five-digit chopping is :

   (A) 3.1416. (B) 3.1415.

   (C) 3.14159. (D) 0.3141.

2. If t is the largest non-negative integer for which $\frac{|p - p^*|}{|p|} \leq 5 \times 10^{-t}$, then t is called :

   (A) Error bound. (B) Five-digit chopping.

   (C) Significant digits. (D) None of these.

3. For $x^3 - 7x^2 + 14x - 6 = 0$ on [0, 1], using Bisection method, p_3 lies in the interval :

   (A) (0.5, 0.75). (B) (0.75, 1).

   (C) (0.5, 0.625). (D) None of these.

4. Find the first approximation of $f(x) = x^3 + x - 1 = 0$ by Newton's method :

   (A) 0.5. (B) 0.45.

   (C) 0.24. (D) 0.75.

5. The linear Lagrange interpolating polynomial that passes through the points (2, 4) and (5, 1) is :

   (A) $-x + 6$. (B) $-x + 5$.

   (C) $x - 6$. (D) $x - 5$.

6. For $f(x) = \tan x$, and $x_0 = 0$, and $x_1 = 0.6$, find interpolation polynomial of degree at most one :

   (A) $1.031121x + 1$. (B) $0.031121x - 1$.

   (C) $1.031121x$. (D) $0.031121x - 1$.

7. The Trapezoidal rule is used for approximation of :

   (A) Differentiation. (B) Integration.

   (C) Both (A) and (B). (D) None of the above.

8. The three-point midpoint formula is used for the approximation of :

(A) Differentiation. (B) Integration.

(C) Both (A) and (B). (D) None of the above.

9. The approximation $\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0)+f(x_1)\right]$ is the :

(A) Newton's formula. (B) Simpson's 1/3rd rule.

(C) Trapezoidal rule. (D) Simpson's 3/8th rule.

10. By Trapezoidal rule $\int_0^2 (x+1)^{-1}\,dx$ is :

(A) 1.333. (B) 1.111.

(C) 1.099. (D) 0.099.

11. By Simpson's rule $\int_0^2 e^x\,dx$ is :

(A) 6.389. (B) 4.164.

(C) 8.389. (D) 6.421.

12. What is the Lipschitz constant for the function $f(t, y) = \cos(yt)$ on $D = \{(t, y)\ 0 \le t \le 1, -\infty < y < \infty\}$?

(A) 0. (B) 1.

(C) 2. (D) 3.

13. What is the Lipschitz constant for the function $f(t, y) = ty$ on $D = \{(t, y)\ 0 \le t \le 1, -\infty < y < \infty\}$?

(A) 0. (B) 3.

(C) 2. (D) 1.

14. The difference-equation method resulting from replacing $T^{(2)}(t, y)$ in Taylor's method of order two by $f(t + (h/2), y + (h/2) f(t, y))$ is known as :

(A) Modified Euler method.
(B) Milne Thomson method.
(C) Midpoint method.
(D) Adams Bashforth method.

15. Find approximation to the solution at $y(0.2)$ using Runge-Kutta method of order four for the IVP $y' = y - t^2 + 1, y(0) = 0.5$?

(A) 1.21407.
(B) 1.82929.
(C) 0.82929.
(D) 0.21407.

D 10668 (Pages : 3) Name

D 10668

Reg. No........

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2(

(CBCSS—UG)

ˋen

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1, 2].

2. Determine fixed points of the function $g(x) = x^2 - 2$.

3. Write the equation of Lagrange's interpolating polynomial through (x_0, y_0) and (x_1, y_1).

4. State three point end point formula of differentiation.

5. Using Trapezoidal rule find $\int_0^2 x^2 \, dx$.

6. Show that $f(t, y) = t|y|$ satisfies a Lipschitz condition on the interval $D = \{(t, y)/1 \leq t \leq 2$ and $-3 \leq y \leq 4\}$.

7. Define a convex set.

8. For all $x \geq -1$ and any positive m show that $0 \leq (1+x)^m \leq e^{mx}$.

9. When is the initial value problem $\frac{dy}{dt} = f(t, y), \ a \leq t \leq b, \ y(a) = \alpha$ well posed.

10. What is the degree of accuracy or precision of a quadrature formula ?

Turn over

11. Write Newton's Forward difference formula.

12. Set up Newton-Raphson formula for computing $\sqrt{N}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find a root of $f(x) = x^3 - 3x - 5 = 0$ correct to 3 decimal places using Newton-Raphson method. Start with $x_0 = 3$.

14. Using Lagrange's interpolation formula find y (10) if :

x	:	5	6	9	11
y	:	12	13	14	16

15. Using Newton's forward interpolation formula find the cubic polynomial for the data :

x	:	0	1	2	3
y	:	1	2	1	10

16. Approximate $\int_1^2 \frac{1}{x}\, dx$ using Simpson's $\frac{3}{8}$ th rule with step value $h = 0.25$

17. Using Second derivative midpoint formula approximate $f^{11}(1.3)$ if $f(x) = 3xe^x - \cos x$ with $h = 0.1$. Given :

x	:	1.2	1.29	1.30	1.31	1.40
y	:	11.59006	13.78176	14.04276	14.30741	16.86187

18. Use Euler's method to find approximate solution for the initial value problem $y^1 = 1 + \frac{y}{t}$, $1 \le t \le 2$, $y(1) = 2$ with $h = 0.25$.

19. Use Newton's Backward difference formula to construct interpolating polynomial of degree 1 if $f(-0.75) = -.07181250$, $f(-0.5) = -.02475000$, $f(-.25) = .33493750$, $f(0) = 1.10100000$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Find by the method of Regula Falsi a root of the equation $x^3 + x^2 - 3x - 3 = 0$ lying between 1 and 2.

21. Use the Modified Euler method to approximate the solutions to the IVP $y^1 = \frac{1+t}{1+y}, 1 \leq t \leq 2,$ $y(1) = 2$ with $h = 0.5$.

(1 × 11 = 11 marks)

D 10667-A (Pages : 6) Name.......................................

Reg. No...................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS–UG)

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5B 06—BASIC ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Which one of the following statements is true ?

   (A) If $n \in \mathbb{N}$, then there is an injection from N_n into N.

   (B) If $n \in \mathbb{N}$, then there doesn't exist an injection from N_n into N.

   (C) If $n \in \mathbb{N}$, then there is a bijection from N_n into N.

   (D) None of the above options.

2. A set S is denumerable if —————.

   (A) There exists an injection from S into N.

   (B) There exists a surjection from N onto S.

   (C) There exists a bijection from N onto S.

   (D) None of the above options.

3. The set A of all real numbers x such that $3x + 2 \leq 6$ is given by —————

   (A) $A = \left\{x \in R : x \leq \frac{2}{3}\right\}$.

   (B) $A = \left\{x \in R : x \geq \frac{4}{3}\right\}$.

   (C) $A = \left\{x \in R : x \leq \frac{4}{3}\right\}$.

   (D) None of the above options.

4. The set $S = \left\{x \in R : x^2 + 2x > 3\right\}$ is —————.

   (A) $S = \{x \in R : x > 1\} \cup \{x \in R : x < -3\}$.

   (B) $S = \{x \in R : x > 1\} \cup \{x \in R : x < -2\}$.

   (C) $S = \{x \in R : x > 2\} \cup \{x \in R : x < -3\}$.

   (D) None of the above options.

5. Which one of the following is not a sequence ?

(A) $(1,2,1,2,1,\ldots)$.

(B) $(1,3,5,7,\ldots,(2n-1),\ldots)$.

(C) $\left(\frac{1}{2},\frac{1}{6},\frac{1}{12},\frac{1}{18},\frac{1}{24},\ldots\right)$.

(D) $(1,2,3,4)$.

6. The first five terms of the sequence (x_n), where $x_n = \frac{1}{n(n+1)}$, are ————.

(A) 1, 1, 1, 1, 1.

(B) $\frac{1}{2},\frac{1}{3},\frac{1}{6},\frac{1}{7},\frac{1}{8}$.

(C) $\frac{1}{2},\frac{1}{6},\frac{1}{12},\frac{1}{18},\frac{1}{24},\ldots$

(D) None of the above options.

7. n^{th} term of the sequence $\left(0,\frac{1}{2},\frac{2}{3},\frac{3}{4},\ldots\right)$ is ————.

(A) $\frac{n+1}{n}$. (B) $\frac{n-1}{n}$.

(C) $\frac{n}{n-1}$. (D) $\frac{n}{n+1}$.

8. The sequence $\{\sqrt{n}\}$ ————.

(A) Converges to 0. (B) Converges to 1.

(C) Converges to $\sqrt{2}$. (D) Diverges.

9. The Fibonacci sequence is given by ————————.

(A) $(1,2,2,3,5,8,13,21,34,55,\ldots)$.

(B) $(1,1,2,3,5,8,13,21,34,55,\ldots)$.

(C) $(1,1,2,,4,5,8,13,21,34,55,\ldots)$.

(D) $(1,2,3,4,5,8,13,21,34,55,\ldots)$.

10. If X and Y are the sequences $X=(2,4,6,\ldots,2n,\ldots), Y=\left(\frac{1}{1},\frac{1}{2},\frac{1}{3},\ldots,\frac{1}{n},\ldots\right)$, then $X-Y=$ ————————.

(A) $\left(\frac{1}{2},\frac{7}{2},\frac{17}{3},\ldots,\frac{2n^2-1}{n},\ldots\right)$.

(B) $\left(\frac{1}{1},\frac{7}{2},\frac{17}{3},\ldots,\frac{2n^2-1}{n},\ldots\right)$.

(C) $\left(\frac{1}{1},\frac{3}{2},\frac{7}{3},\ldots,\frac{2n^2-1}{n},\ldots\right)$.

(D) None of the above options.

11. $\lim\left(\frac{2n}{n^2+1}\right)=$ ————————.

(A) 0.

(B) 1.

(C) 2.

(D) 3.

12. $\lim\left(\frac{\sin n}{n}\right)=$ ————————.

(A) 1.

(B) 2.

(C) 3.

(D) None of the above options.

13. The sequence $S = (\sin n)$ ————————.

(A) Is divergent.
(B) Converges to 0.
(C) Converges to 1.
(D) None of the above options.

14. The sequence $\left(\sin \frac{n\pi}{4}\right)$

(A) Is divergent.
(B) Converges to 0.
(C) Converges to 1.
(D) None of the above options.

15. $\left(1 + \frac{1}{2!} + \ldots + \frac{1}{n!}\right)$ ————————.

(A) Is a Cauchy sequence.
(B) Not be Cauchy sequence.
(C) Not convergent.
(D) None of the above options.

16. Which one of the following is a false statement ?

(A) There are bounded sequences that are not Cauchy.

(B) $\left((-1)^n\right)$ is not a Cauchy sequence.

(C) If (x_n) is a Cauchy sequence such that x_n is an integer for all $n \in \mathrm{N}$, then (x_n) is ultimately constant.

(D) If (x_n) is a Cauchy sequence such that x_n is an integer for all $n \in \mathrm{N}$, then (x_n) is a constant sequence.

17. Let $\mathrm{F} \subseteq \mathrm{R}$ be such that if $\mathrm{X} = (x_n)$ is any convergent sequence of elements in F, then lim X belongs to F, then ————————.

(A) F is a closed set.
(B) F is an open set.
(C) F is neither closed nor open.
(D) F is both open and closed.

18. The Cantor set ————————.

(A) Has only finite number of points.
(B) Has countable number of points.
(C) Is denumerable.
(D) Is uncountable.

19. If $z = -\sqrt{3} - i$, then $z^3 =$ ________.

(A) $8i$.
(B) $-8i$.
(C) $-7i$.
(D) $7i$.

20. If $z = 2 + \pi i$, then $e^z =$ ________.

(A) $-3e^2$.
(B) $2e^2$.
(C) $2e^3$.
(D) $-e^2$.

D 10667 (Pages : 2) Name.....................................

Reg. No...............................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS–UG)

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Is the union of two disjoint denumerable sets denumerable ?
2. If $a, b \in \mathbb{R}$ with $ab = 0$, then prove that either $a = 0$ or $b = 0$.
3. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then show that $a = 0$.
4. Find all real numbers x satisfying the inequality $x^2 > 3x + 4$.
5. If $0 < c < 1$, then show that $0 < c^2 < c < 1$.
6. If x and $y \in \mathbb{R}$ with $x < y$ prove that there exists an irrational number z such that $x < z < y$.
7. State characterization theorem for intervals.
8. Test the convergence of $\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \dots\right)$.
9. Show that every convergent sequence is a Cauchy sequence.
10. Define Supremum of a set and give example of a set which has no Supremum.
11. What can be said about the complex number z if $z = -\bar{z}$.
12. Find modulus of the complex number $z = -9i$.
13. Find real and imaginary parts of the complex function $f(z) = \bar{z}$ as functions of r and θ.
14. State nested interval property.
15. Write the equation of (a) a closed disk of radius ρ centred at z_0 ; (b) equation of a circle with centre z_0 and radius ρ.

(10 × 3 = 30 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. State and prove Cantor's theorem.

17. Prove that there does not exist a rational number r such that $r^2 = 2$.

18. Solve the inequality $|2x-1| \leq x+1$.

19. Let S be a non-empty set in $\mathbb{R}$, that is bounded above. Prove that Sup $(a + S) = a$ + Sup S.

20. State and prove Archimedean property.

21. Prove that a sequence in $\mathbb{R}$ can have atmost one limit.

22. Find the image of the half plane $\text{Re}\, z \geq 2$ under the mapping $W = iZ$.

23. Prove that $|z_1 - z_2| \geq \left||z_1| - |z_2|\right|$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) State and prove Arithmetic-geometric inequality.

    (b) Let $a,b,c \in \mathbb{R}$. Then if $ab < 0$ then show that either $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

    (c) If $1 < C$, then show that $1 < C < C^2$.

25. (a) Prove that every contractive sequence is a Cauchy sequence.

    (b) Prove that if a sequence X of real numbers converges to a real number x, then any subsequence of X also converge to x.

26. (a) The polynomial equation $x^3 - 7x + 2 = 0$ has a solution between 0 and 1. Use an approximate contractive sequence to calculate the solution correct to 4 decimal places.

    (b) Show that $\lim\left(n^{\frac{1}{n}}\right) = 1$.

27. (a) Find an upperbound for $\left|\dfrac{1}{z^4 - 5z + 1}\right|$ if $|z| = 2$.

    (b) Find the image of the vertical strip $2 \leq \text{Re}\, Z < 3$ under the mapping $f(Z) = 3Z$.

    (c) Find the domain of $f(z) = \dfrac{iz}{|z| - 1}$.

(2 × 10 = 20 marks)

D 10666-A (Pages : 4) Name..

Reg. No..

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. If H_1 and H_2 are two right cosets of subgroup H then :

   (A) $H_1 \cap H_2 = \phi\ H_1 = H_2$.
   (B) $H_1 \cap H_2 \neq \phi$.
   (C) $H_1 \cup H_2 = \phi$.
   (D) $H_1 \neq H_2$ and $H_1 \cap H_2 \neq \phi$.

2. (i) Every isomorphic image of a cyclic group is cyclic.

   (ii) Every homomorphic image of cyclic group is cyclic.

   (A) Both (i) and (ii) are true.
   (B) Both (i) and (ii) are false.
   (C) (i) is true only.
   (D) (ii) is true only.

3. E is set of integers under ordinary addition and multiplication, then E is a ring, E is also a ——————.

   (A) Commutative ring.
   (B) Integral domain.
   (C) Both (A) and (B).
   (D) None of these.

4. The set of all rational numbers of the form $3^m . 6^m$; m, n are integers :

   (A) Forms a group under multiplication.
   (B) Forms a group under addition.
   (C) Forms a group under division.
   (D) None of the above.

5. A group contains two elements a and b such that $|a| = 4, |b| = 2$, and $a^3 . b = ba$. Then $|ab| =$

   (A) 2.
   (B) 4.
   (C) 6.
   (D) 8.

6. Let G_n be a cyclic group of order n. Which of the following direct product is not cyclic ?

   (A) $G_{22} \times G_{31}$.
   (B) $G_{222} \times G_{333}$.
   (C) $G_{17} \times G_{11}$.
   (D) $G_{17} \times G_{11} \times G_5$.

7. What is the largest order of an element in the group of permutations of 5 objects ?

(A) 5.
(B) 6.
(C) 12.
(D) 60.

8. The set of all generators of a cyclic group $G = \langle a \rangle$ of order 8 is :

(A) $\{a^2, a^4, a^6\}$.
(B) $\{a^1, a^3, a^5, a^7\}$.
(C) $\{a^4, a^8\}$.
(D) $\{a^3, a^5, a^7\}$.

9. Number of self-inverse elements of D_4 :

(A) 0.
(B) 2.
(C) 4.
(D) 6.

10. Identity element of the group R under addition is :

(A) 1.
(B) 2.
(C) 0.
(D) -1.

11. $\{1, i, -i, -1\}$ is ————.

(A) Semigroup.
(B) Subgroup.
(C) Cyclic group.
(D) Abelian group.

12. Let A, B be non-empty sets and $f : A \to B$ be a permutation. Then ————.

(A) f is bijective and $A = B$.
(B) f is one one and $A \neq B$.
(C) f is bijective and $A \neq B$.
(D) f is onto and $A \neq B$.

13. The number of elements in A_6 = ————.

(A) 6.
(B) 720.
(C) 360.
(D) 2^6.

14. Let H be a subgroup of a group G and $a, b \in G$. Then $b \in aH$ if and only if :

(A) $ab \in H$.
(B) $ab^{-1} \in H$.
(C) $a^{-1}b \in H$.
(D) None of these.

15. If G be a cyclic group of order 8 with generator x then another generator of G be :

(A) x^5.
(B) x^4.
(C) x^6.
(D) x^2.

16. Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is :

(A) 0.
(B) 1.
(C) 3.
(D) 4.

17. If G is commutative group then $(ab)^n =$ .... $\forall n \in Z$.

(A) ab.
(B) ba.
(C) $a^n b^n$.
(D) $a^n b^m$.

18. If $(x-a)$ is a factor of $f(x)$ then $f(a)$:

(A) a.
(B) 0.
(C) 1.
(D) $a-1$.

19. Sum of the roots of the equation $x^5 - 5x^3 + x + 1 = 0$ is given by :

(A) 0.
(B) 5.
(C) -1.
(D) None of these.

20. A polynomial equation in x of degree n always has :

(A) n distinct roots.
(B) n real roots.
(C) n imaginary roots.
(D) At most one root.

D 10666 (Pages : 3) Name.......................................

Reg. No.......................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.

2. Factorize into linear factors the polynomial $x^4 - 1$.

3. Write a cubic equation with roots $1,\ 1+i,\ 1-i$.

4. State Identity theorem.

5. How many real roots has the equation $x^4 - 4ax + b = 0$.

6. Make addition and multiplication tables for $\mathbb{Z}_2$.

7. Check whether the relation on $\mathbb{R}$ defined by $a \sim b$ if $a - b \in \mathbb{Q}$ is an equivalence relation.

8. Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Compute $\sigma\tau$ and $\tau\sigma$.

9. Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

10. Write a subgroup of $(\mathbb{Z}, +)$.

Turn over

11. Check whether $\mathbb{Z} \times \mathbb{Z}$ is cyclic.

12. Find order of the permutation $(1, 3)(2, 6)(1, 4, 5)$.

13. Let $\Phi : G_1 \to G_2$ be a group homomorphism. Show that $\Phi(e) = e'$ where e and e' are identity elements of G_1 and G_2 respectively.

14. Define a Ring.

15. Give example of an integral domain.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Solve $x^5 - 3x^4 + 4x^3 - 4x + 4$ having the root $1 + i$.

17. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ whose roots are a, b, c with $a + b = 0$.

18. Find an upper limit of the positive roots of the equation $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.

19. Prove that set of all even permutations of S_n is a subgroup of S_n.

20. Define * on $\mathbb{Z}$ by $a * b = a - b$. Check whether $(\mathbb{Z}, *)$ is a group.

21. Check whether $\mathbb{Z}_n$ is cyclic.

22. Draw the subgroup diagram of $\mathbb{Z}_{36}$.

23. Let G_1 and G_2 be groups and let $\Phi : G_1 \to G_2$ be a function such that $\Phi(a\,b) = \Phi(a)\,\Phi(b)$ for all $a, b \in G$. Prove that Φ is $1 - 1$ if and only if $\Phi(x) = e$ implies that $x = e$ for all $x \in G_1$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Examine whether $x^4 - x^3 - x^2 + 19x - 42 = 0$ has integral roots or not.

25. Solve $x^3 - 6x - 6 = 0$ by Cardan's method.

26. Let G be a cyclic group. Show that :

    (a) If G is infinite then $G \cong \mathbb{Z}$.

    (b) If $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Lagrange's theorem.

(2 × 10 = 20 marks)

D 10236-A (Pages : 3) Name..

Reg. No.....................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5D 19—MATHEMATICS FOR SOCIAL SCIENCES

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 10	Maximum : 10 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 10.
2. The candidate should check that the question paper supplied to him/her contains all the 10 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5D 19—MATHEMATICS FOR SOCIAL SCIENCES

(Multiple Choice Questions for SDE Candidates)

1. If $f(x) = x^2 + 2x + 6$ then $f(0)$ is ————.

(A) 6. (B) 9.

(C) 0. (D) 5.

2. The limit of $f(x) = x^2 + x(x-3)$ as $x \to 3$ is ————.

(A) 3. (B) 9.

(C) 0. (D) – 3.

3. If $f(x) = (2x+5)^5$ then $f(x)$ is ————.

(A) $\frac{5}{(2x+5)^5}$. (B) $5(2x+5)^4$.

(C) $\frac{(2x+5)^6}{12}$. (D) $10(2x+5)^4$.

4. If $f(x) = 6x^3$ then $f''(x)$ is ————.

(A) $\frac{5}{x^4}$. (B) $18x^2$.

(C) $36\,x$. (D) 36.

5. At $x = 2$ the function $f(x) = x + 5$ is ————.

(A) Increasing. (B) Decreasing.

(C) Inflection. (D) Slope = 0.

6. The value of $\log_2 16$ is equivalent to ————.

(A) 2 (B) 0.

(C) 4. (D) 8.

7. Derivative of $(x) = e^{2x^3}$ is ————.

(A) e^{6x^3}.

(B) $6x^2e^{2x^3}$.

(C) e^{2x^3}.

(D) $\frac{e^{2x^3}}{6x}$.

8. The value of $\int 6e^{3x}dx$ is ————.

(A) $2e^{3x} + c$.

(B) $\frac{6}{e^{3x}} + c$.

(C) $18e^{3x} + c$.

(D) $\frac{6e^{4x}}{4} + c$.

9. If $z = e^{x^2y}$ then $\frac{\partial z}{\partial x}$ is ————.

(A) $x^2e^{x^2y}$.

(B) xye^{x^2y}.

(C) $2xye^{x^2y}$.

(D) $\frac{x^2y}{3}ex^2y$.

10. Using L'Hospital's rule the limit of $f(x) = \frac{x^2 - 7x}{x^3 + 2x}$ as $x \to 0$ is ————.

(A) 0.

(B) 2.

(C) $\frac{-7}{2}$.

(D) 7.

D 10236 (Pages : 2) Name.................................

Reg. No..............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5D 19—MATHEMATICS FOR SOCIAL SCIENCES

Time : Two Hours Maximum : 40 Marks

Section A

Answer all the **six** *questions.*
Each question carries 1 *mark.*

1. Solve : $\frac{x}{6} - 5 = \frac{x}{9} + 1$.

2. Find $\lim_{x \to 4} \sqrt{2x^3 - 7}$.

3. Define concavity and convexity.

4. Find the partial derivative $\frac{\partial z}{\partial y}$ if $z = 8x^2 + 14xy + 5y^2$.

5. Find the marginal cost at Q = 3 if the total cost function TC = $3Q^2 + 7Q + 12$.

6. Convert the logarithm $\log_a y = 5x$ into equivalent natural exponential form.

(6 × 1 = 6 marks)

Section B

Answer any **five** *out of seven questions.*
Each question carries 2 *marks.*

7. Verify whether the function $f(x) = \frac{x^2 + 3x + 12}{x - 3}$ is continuous at $x = 4$.

8. Use the point-slope formula to derive the equation of the line passing through (3, 11) and having slope – 4.

9. Find the derivative of $y = (3x^3 + 8)^5$.

10. Solve $7 \log_e x - 2.6 = 10$.

11. Integrate $\sqrt{3x^2 - 4}\,(6x)$ with respect to x.

Turn over

12. Evaluate $\int_1^3 (6x^2 + 5)\, dx$.

13. The marginal cost function of a firm is given by MC = $3{,}000e^{0.3x} + 50$, when x is quantity produced. If fixed cost is Rs. 80,000, find the total cost function of the firm.

(5 × 2 = 10 marks)

Section C

Answer any **three** *out of five questions.*
Each question carries 4 *marks.*

14. If $y = x^{\log x}$, find $\frac{dy}{dx}$.

15. Evaluate $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x+4} - \sqrt{x}\right)$.

16. Find the cross partial derivatives z_{xy} and z_{yx} for the function $z = 10(9x - 4y)^5$.

17. Find the relative extrema for the function $f(x) = -9x^2 + 126x - 45$.

18. The marginal revenue function of a product is given by MR = $500 - 0.01x$ and the marginal cost function is given by MC = $100 + 0.006x$. The fixed cost is Rs. 1,50,000. Find the profit function.

(3 × 4 = 12 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 6 *marks.*

19. Use integration by parts to find the indefinite integral $\int 30x\sqrt{9+x}\, dx$.

20. A firm has the demand function P = 100 – 0.01Q and total cost function TC = 50Q + 30,000 where P is the price and Q is the number of units. How many units are to be manufactured to maximize the profit.

21. (a) Evaluate $\int \frac{xe^x}{(x+1)^2}\, dx$.

(b) Find the equation of the line passing through the point (3, 1) and perpendicular to the line $2x + 7y - 5 = 0$.

(2 × 6 = 12 marks)

D 10235 (Pages : 3) Name.....................................

Reg. No.....................................

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5D 18—MATHEMATICS FOR NATURAL SCIENCES

Time : Two Hours Maximum : 40 Marks

Section A

Answer all the **six** *questions.*
Each question carries 1 *mark.*

1. Define discrete data and continuous data with an example.
2. Find the log to the base 8 of 4096.
3. Define any two measures of central tendency.
4. Find the median of the set of numbers 5, 5, 7, 9, 11, 12, 15, and 18.
5. Find the quadratic mean of the numbers 3, 5, 6, 6, 7, 10, and 12.
6. Define skewness of a distribution.

(6 × 1 = 6 marks)

Section B

Answer any **five** *out of seven questions.*
Each question carries 2 *marks.*

7. Solve the logarithmic equation $\ln(x)^2 - 1 = 0$.
8. Describe two graphic representations of frequency distributions.
9. Ten measurements of the diameter of a cylinder were recorded by a scientist as 3.88, 4.09, 3.92, 3.97, 4.02, 3.95, 4.03, 3.92, 3.98, and 4.06 centimeters (cm). Find the arithmetic mean of the measurements.
10. Find the mean and mode for the set of numbers 3, 5, 2, 6, 5, 9, 5, 2, 8, and 6.
11. Find the second and third moments of the set of numbers 2, 3, 7, 8, 10.
12. Find the standard deviation of the set of numbers 9, 3, 8, 8, 9, 8, 9, 18.
13. Prove that $m_2 = m_2' - m_1'^2$.

(5 × 2 = 10 marks)

Turn over

Section C

Answer any **three** *out of five questions.*
Each question carries 4 *marks.*

14. Solve the logarithmic equation $\log(6y - 7) + \log y = \log 5$.

15. The smallest of 150 measurements is 5.18 in, and the largest is 7.44 in. Determine a suitable set of : (a) class intervals ; (b) class boundaries ; and (c) class marks that might be used in forming a frequency distribution of these measurements.

16. Four groups of students, consisting of 15, 20, 10, and 18 individuals, reported mean weights of 162, 148, 153, and 140 pounds (lb), respectively. Find the mean weight of all the students.

17. Prove that the sum of the deviations of $X_1, X_2, \ldots, X_N$ from their mean $\bar{X}$ is equal to zero.

18. The bacterial count in a certain culture increased from 1000 to 4000 in 3 days. What was the average percentage increase per day ?

(3 × 4 = 12 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 6 *marks.*

19. The final grades in mathematics of 80 students at State University are recorded in the accompanying table :

68	84	75	82	68	90	62	88	76	93
73	79	88	73	60	93	71	59	85	75
61	65	75	87	74	62	95	78	63	72
66	78	82	75	94	77	69	74	68	60
96	78	89	61	75	95	60	79	83	71
79	62	67	97	78	85	76	65	71	75
65	80	73	57	88	78	62	76	53	74
86	67	73	81	72	63	76	75	85	77

With reference to this table, find :

(a) The highest grade.

(b) The lowest grade.

(c) The range.

(d) The grades of the five highest-ranking students.

(e) The grades of the five lowest-ranking students.

(f) The grade of the student ranking tenth highest.

(g) The number of students who received grades of 75 or higher.

(h) The number of students who received grades below 85.

(i) The percentage of students who received grades higher than 65 but not higher than 85.

(j) The grades that did not appear at all.

20. Find : (a) the quartiles Q1, Q2, and Q3 and (b) the deciles D1, D2,...,D9 for the wages of the 65 employees at the P&R Company.

Wages	Frequency
250.00 - 259.99	8
260.00 - 269.99	10
270.00 - 279.99	16
280.00 - 289.99	14
290.00 - 299.99	10
300.00 - 309.99	5
310.00 - 319.99	2
	Total = 65

21. Find the moment co-efficient of skewness, a3 for the height distribution of students at XYZ University :

Height (in)	Number of Students
60 - 62	5
63 - 65	18
66 - 68	42
69 - 71	27
72 - 74	8
	Total = 100

(2 × 6 = 12 marks)

D 10233-A (Pages : 4) Name.....................................

Reg. No.....................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 30 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

1. The degree of the differential equator $(y')^{3/2} = y''$ is ?

(A) 0. (B) 1.

(C) 2. (D) None of these.

2. The integral curves of the differential equation $y' = 1$ are ?

(A) $y = x + c$. (B) $y = x^2 + c$.

(C) $y = x^3 + c$. (D) $y = x + 1$.

3. An integrating factor of the differential equation $ty' + 2y = 4t^2$ is ?

(A) t^3. (B) t^4.

(C) t^2. (D) None of these.

4. A homogeneous differential equation $\frac{dy}{dx} = f\left(y/x\right)$ can be converted to a variable separable equator using a transformation :

(A) $y = vx$. (B) $y^2 = vx$.

(C) $y = vx^2$. (D) $y = v^2x$.

5. The differential equation $\left(6xy^2 + 4x^3y\right)dx + \left(6x^2y + x^4 + e^y\right)dy = 0$ is ?

(A) A separable equation. (B) A linear equation.

(C) An exact equation. (D) A homogeneous linear equation.

6. An integrating factor of the differential equation $3\left(x^2 + y^2\right)dx + x\left(x^2 + 3y^2 + 6ydy = 0\right)$ is :

(A) e^{-y}. (B) e^2y.

(C) e^y. (D) e^{-2y}.

7. The solution of the differential equation $y' = y^2, y(0) = 1$ exists in the region :

(A) $(0, \infty)$. (B) $(-\infty, 0)$.

(C) $(-\infty, 1)$. (D) $(-\infty, \infty)$.

8. If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ is a function of y only, then an integrating factor of the differential equation $Mdx + Ndy = 0$ is :

(A) $\mu(x) = \exp\left[\int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\right].$

(B) $\mu(x) = \exp\left[\int \frac{1}{M}\left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right) dy\right].$

(C) $\mu(x) = \int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy.$

(D) $\mu(x) = \int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy.$

9. An integrating factor of the differential equation $\frac{dx}{dy} + P(x) = Q(x)$ is :

(A) $e^{\int p dx}.$

(B) $e^{-\int p dx}.$

(C) $e^{\int p^2 dx}.$

(D) $e^{\int (P+Q) dx}.$

10. An integrating factor of the differential equator $\frac{dx}{dy} + Px = Q$ where P and Q are functions of y alone is :

(A) $e^{\int p dy}.$

(B) $e^{-\int p dy}.$

(C) $e^{\int p dx}.$

(D) $e^{\int p^2 dy}.$

11. The initial value problem $y' = y^{1/3}, y(0) = 0, t \geq 0$:

(A) A unique solution.

(B) Infinitely many solutions.

(C) No solution.

(D) Two solutions.

12. The general solution of the differential equation $3(x^2 + y^2)dx + x(x^2 + 3y^2 + 6ydy = 0)$ is :

(A) $x^3 e^{-y} + 3xy^2 e^y = c.$

(B) $x^3 e^y + 3xy^2 e^y = c.$

(C) $x^2 e^y + 3x^2y^2 e^y = c.$

(D) $xe^y + ye^y = c.$

13. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the linear differential equation $a_0(x)y'' + a_1(x)y' + a_2 + (x)y = 0$ then :

(A) $y_1(x)y_2(x)$ is also a solution.

(B) $y_1(x) + y_2(x)$ is also a solution.

(C) $y_1(x)/y_2(x)$ is also a solution.

(D) $y_1^2(x) + y_2^2(x)$ is also a solution.

14. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ then the Wronskian $w(y_1, y_2)$ is :

(A) 1.
(B) 0.
(C) 2.
(D) –1.

15. The general solution of the differential equation $\left(D^2 - 4D + 4\right)y = 0$ is :

(A) $(c_0 + c_1x)e^{2x}$.
(B) $(c_0 - c_1x)e^{2x}$.
(C) $c_1e^x\ c_2e^{-2x}$.
(D) None of these.

16. The Laplace transform of e^{at} is :

(A) $\frac{1}{s-a}$.
(B) $\frac{1}{s+a}$.
(C) $\frac{1}{s^2-a^2}$.
(D) $\frac{1}{s^2-a^2}$.

17. The Laplace transform of cos at is :

(A) $\frac{s}{s^2+a^2}$.
(B) $\frac{1}{s^2+a^2}$.
(C) $\frac{1}{s^2-a^2}$.
(D) $\frac{s}{s^2-a^2}$.

18. If $L\{f(t)\} = F(s)$, then $L\{f(at)\} =$

(A) $\frac{1}{a}f(s/a)$.
(B) $F(s/a)$.
(C) $F(a/s)$.
(D) $F(s)$.

19. The Laplace transform of the delta function is :

(A) e^{-as}.
(B) e^{as}.
(C) $e^{as/s}$.
(D) $e^{-as/s}$.

20. $\int_0^\infty \frac{\sin t}{t}\,dt =$

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{8}$.
(C) $\frac{\pi}{2}$
(D) None of these.

D 10233 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Prove that the product of two odd functions is an even function.
2. Prove $L^{-1}\{1\}$.
3. Write down the differential equation whose solution is $y = c_1 e^{5t} + c_2 e^{-2t}$.
4. Evaluate $W\left[e^{\mu \cos \lambda t}, e^{\mu \sin \lambda t}\right]$.
5. Compute $L\left\{t^2 e^{\lambda t}\right\}$.
6. Find the integrating factor of $(x-2)(x+1)\dfrac{dy}{dx} + 3y = x$.
7. Solve the system $\dfrac{dy}{dt} - x = 0, \dfrac{dx}{dt} - y = 0$.
8. Find the fundamental solutions of $y'' + 25y = t^{-1/2}$.
9. Find the value of b_n in the Fourier sine series expansion of 2π-periodic function $f(x) = -x, x \in [-\pi, \pi]$.
10. Write one dimensional heat equation with all the assumptions involved.
11. What do you mean by an exact differential equation ? Give an example.
12. Find the complementary function corresponding to $y'' - 2y' + 2y = t$.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Convert $y'' + 2y' = 0$ into a system of first order equations.

Turn over

14 Find the Fourier cosine series for the 2π periodic function $f(x) = x, x \in [-\pi, \pi]$.

15 Find the integrating factor for $(2x + 3y)dx + (2x - 3y)dy = 0$.

16 Find the inverse Laplace transform of $\log((s - a)/(s - b))$

17 Define unit step function and find its Laplace transform.

18 Solve $t^2x'' - 2tx' - 3x = 0$.

19 Write the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.

20 Show that the inverse Laplace transform is linear.

21 State Abel's theorem.

22 Solve $\frac{dy}{dx} = (3x + 2y + 1)^2$.

23 Evaluate $L\left\{te^t \cos 2t\right\}$.

24 Find the second order p.d.e. for which $y = \phi(x + at) + \psi(x - at)$ is a solution.

25 Solve the system : $\frac{dy}{dt} = x - y, \frac{dx}{dt} = x + y$.

26 Solve $y' - 2y = 0$ using Laplace transform.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27 Express the function $f(t) = \begin{cases} t\sin t, & \text{if } 0 \le t < \pi/2 \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0, & \text{elsewhere} \end{cases}$ in terms of combination of unit step functions and hence find its Laplace transform.

28. Evaluate the Laplace inverse transforms of $4 - \cot^{-1}(s/a)$ and $\frac{1}{\left(s^2 - 5s + 6\right)^2}$.

29. Find the solution by the checking the exactness of $\left(3y^2 - 2xy + 2\right)dx + \left(6xy - x^2 + y^2\right)dy = 0$.

30. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

31. A ball with mass 0.15 kg. is thrown upward with initial velocity 20 m/s from the roof of a building 30 m. high. Neglect air resistance (a) Find the maximum height above the ground that the ball reaches ; (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

32. Find the Fourier cosine series for the function $f(t) = \pi|t - \pi|, t \in [0, \pi]$.

33. Find the solution of the heat conduction problem :

$$25u_{xx} = u_t, 0 < x < 1, t > 0; u(0,t) = 0, u(1,t) = 0, t > 0; u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

34. State and prove Convolution theorem for Laplace transforms.

35. Evaluate (i) $L^{-1}\left(\frac{1-e^{-s}}{s}\right)$ and (ii) get a formula for $L(f(t))$ where $f(t)$ is a periodic function of period T.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 marks.

36. (a) Use method of separation of variables and solve the one-dimensional heat equation completely. State the assumptions involved therein explicity.

(b) Find the solution of the p.d.e. $\frac{\partial^2 u}{\partial y \partial x} = 2x$.

37. (a) Solve the following differential equation in two ways, one of them must be using Laplace transform. $4y'' - y = t, y(0) = 1, y(1) = 0$.

(b) Find the Fourier series of $f(x) = x^2, x \in [-2, 2]$ treating it as a periodic function of period 4.

38. (a) Apply method of variation of parameters to solve : $y'' - y = \sec t$.

(b) Solve $(2x + y + 3)\,dx + (x - 3y + 2)\,dy = 0$.

(2 × 13 = 26 marks)

D 10232-A (Pages : 4) Name.....................................

Reg. No.....................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 30 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. If $A_n = \{n, n+1, n+2, \ldots\}$, then $\bigcap_{n=1}^{\infty} A_n =$ ————————.

   (A) 1. (B) ϕ.

   (C) ∞. (D) n.

2. Consider the function $f(x) = \frac{1}{x^2}, x \neq 0$. Determine the image $f(E)$ where $E = \{x \in R : 1 \leq x \leq 2\}$.

   (A) $[1/4, 1]$. (B) $[1/2, 1]$.

   (C) $(0, 1/4]$. (D) $[0, 1/4]$.

3. If $f(x) = 2x$, and $g(x) = 3x^2 - 1$, then $(f \circ g)(x)$ is ————————.

   (A) $12x^2 - 1$. (B) $12x^2 - 2$.

   (C) $6x^2 - 1$. (D) $6x^2 - 2$.

4. Which of the following is true ?

   (A) $(f \circ g)^{-1}(H) = g^{-1}(f^{-1}(H))$. (B) $(f \circ g)^{-1}(H) = f^{-1}(g^{-1}(H))$.

   (C) $(f \circ g)^{-1}(H) \subseteq g^{-1}(f^{-1}(H))$. (D) $(f \circ g)^{-1}(H) \subseteq f^{-1}(g^{-1}(H))$.

5. For each $n \in N$ let $A_n = \{(n+1)k : k \in N\}$. Then, $A_1 \cap A_2$ is :

   (A) A_3. (B) A_4.

   (C) A_5. (D) A_6.

6. The function $f : \mathbb{R} \to (-1, 1)$ defined by $f(x) = x/\sqrt{x^2 + 1}$ is :

   (A) A surjection but not injection. (B) An injection but not surjection.

   (C) Neither injection nor surjection. (D) A bijection.

7. Which of the following set is not countable ?

(A) $\{1, 2, \ldots\ldots, n\}$

(B) The set $\mathbb{N}$ of natural numbers.

(C) The set $\mathbb{Q}$ of rational numbers.

(D) The interval (0,1).

8. If S = {2, 3, 4} the number of elements in P (S), the power set of S, is :

(A) 3.

(B) 6.

(C) 8.

(D) 9.

9. Which of the following is *not* true ?

(A) If $a > b$ and $c > 0$, then $a + c > b + c$.

(B) If $a > b$ and $c < 0$, then $a + c < b + c$.

(C) If $a > b$ and $c > 0$, then $ac > bc$.

(D) If $a > b$ and $c < 0$, then $ac <$ bc.

10. If $a \in \mathbb{R}$ such that, $0 \le a < \varepsilon$ for every $\varepsilon > 0$ then, :

(A) $a > 0$.

(B) $a \neq 0$.

(C) $a = 0$.

(D) None of these.

11. Let $S = \{x \in \mathbb{R} : x < 2\}$. Then :

(A) Neither sup S nor inf S exist.

(B) Both sup S and inf S exist.

(C) Sup S exists but less than 2.

(D) Sup S equal to 2.

12. If $S = \left\{1 - \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$, then :

(A) Sup S = 2, Inf S = 1/2.

(B) Sup S = 2, Inf S = 0.

(C) Sup S = 1, Inf S = 1/2.

(D) Sup S = 1, Inf S = 0.

13. The binary representation of 3/8 is :

(A) 0. 0111111 . . . .

(B) 0. 0101000 . . . .

(C) 0. 1011111 . . . .

(D) 0.0101111 . . . .

14. The rational number represented by 7.3141414 ———— is :

(A) 7245/990.
(B) 7249/990.
(C) 7241/990.
(D) 7243/990.

15. The sixth term of the Fibonacci sequence is ————.

(A) 5.
(B) 6.
(C) 8.
(D) 13.

16. Limit of the sequence $\left(\frac{3n+2}{2n+1}\right)$ is ————.

(A) 3.
(B) 1/2.
(C) 2.
(D) 3/2.

17. The smallest value of $K(\varepsilon)$ corresponding to $\varepsilon = .01$ for the sequence $\left(\frac{1}{n}\right)$ is ————.

(A) 10.
(B) 50.
(C) 100.
(D) 101.

18. Which of the following is false ?

(A) If (x_n) is a convergent sequence then $\left(x_n^2\right)$ is convergent.

(B) If (x_n) is a convergent sequence, and $x_n \geq 0$ for every n, then $\left(\sqrt{x_n}\right)$ is convergent.

(C) If $\left(x_n^2\right)$ is a convergent sequence then (x_n) is convergent.

(D) If (x_n) is a convergent sequence then $\left(x_n^3\right)$ is convergent.

19. The limit of the sequence $\left(1+\frac{1}{2n}\right)^n$ is :

(A) 1.
(B) ∞.
(C) e.
(D) $\sqrt{e}$.

20. The sequence $\left(4,-2,0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\ \ldots\ \ldots\right)$

(A) Monotone decreasing.
(B) Monotone increasing.
(C) Ultimately Monotone decreasing.
(D) Ultimately Monotone increasing.

D 10232 (Pages : 3) Name.................................

Reg. No..............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 mark.

1. Fill in the blanks : Supremum of the set $S = \{1 - 1/n; n \in N\}$ is ———.
2. Determine the set $A = \{x \in R : |x - 1| < |x|\}$.
3. The set of all real numbers which satisfy the inequality $0 \leq b < \in, \forall \in > 0$, then $b =$ ———.
4. Fill in the blanks : The Supremum property of R states that ———.
5. State the Trichotomy Property of R.
6. Give the condition for a subset of R to be an interval of R.
7. State the general Arithmetic Geometric mean inequality of real numbers.
8. Fill in the blanks : The characterization theorem of open sets states that ———.
9. State the Bernoulli's inequality.
10. If $a > 0$, then $\lim\left(a^{1/n}\right) =$ ———.
11. Fill in the blanks : Arg $(-2\pi) =$ ———.
12. Fill in the blanks : The Exponential form of $-1 - i =$ ———.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Define Supremum an Infimum of a set. Find them for the set $S = \left\{1/2^m - 1/3^n; m, n \in N\right\}$.
14. Show that there doesn't exist a rational number r such that $r^2 = 3$.
15. If $a, b \in R$, then prove that $\left||a| - |b|\right| \leq \left||a - b|\right|$.
16. Prove that a real sequence can have at most one limit.

Turn over

17. If $x \in \mathbb{R}$ then prove that there exists $n_x \in \mathbb{N}$ such that $x < n_x$.

18. Discuss the convergence of the following sequences $X = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$

19. Show directly that a bounded monotonic sequence is a Cauchy sequence.

20. Define Cauchy sequence. Test whether $(1/n)$ is a Cauchy sequence or not.

21. Show by an example that intersection of infinitely many open sets in R need not be open.

22. Discuss the convergence of $X = (x_n)$ define by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging subsequence.

24. Test the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.

25. Express the complex number $(\sqrt{3} + i)^7$ in exponential form.

26. Find the principal value of $(-8i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Prove that the set R of real numbers is uncountable.

28. State and prove the Ratio Test for the convergence of real sequence.

29. Discuss the convergence of the following sequences $X = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$ and (b) $x_n = \left(\frac{1-2}{n}\right)^n$.

30. $X = x_n$ and $Y = y_n$ be sequences of real numbers converges to x and y respectively, then prove that X . Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right) = 1$.

    (b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right)=1$.

32. If $X=(x_n)$ is a real sequence and $X_m=(x_{m+n}:n\in N)$ is the m-tail of X ; $m\in N$, then show that X_m converges to x if and only if X converges to x.

33. Let $X=(x_n)$ be a bounded sequence of real numbers and $x\in R$ has the property that "every converging subsequence of $X=(x_n)$ converges to x". Prove that $X=(x_n)$ converges to x.

34. Prove or disprove the following statement : $\|z_1\|-|z_2|\le|(z_1)|-|(z_2)|, \forall z_1,z_2\in C$.

35. Test the convergence of (x_n) defined by $x_n=1+1/2+1/3+\ldots+1/n$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Show that there exists a positive real number x such that $x^2 = 2$.

37. (a) If $I_n=[a_n,b_n], n\in N$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

(b) Test the convergence of (x_n) defined by $x_n=1+1/2!+1/3!+\ldots+1/n!$.

38. (a) Define a closed set and "cluster point" of a set. Give examples for each of them.

(b) Prove that a subset of R is closed in R if and only if it contains all of its cluster points.

(2 × 13 = 26 marks)

D 10231-A (Pages : 4) Name.....................................

Reg. No.....................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 06—ABSTRACT ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. Which of the following defines a binary operation on Z^+ ?

   (A) $a * b = a - b$.

   (B) $a * b = c$, where c is the smallest integer greater than both a and b.

   (C) $a * b = c$, where c is at least 5 more than $a + b$.

   (D) $a * b = c$, where c is the largest integer less than the product of a and b.

2. Under which of the following binary operation, the set of positive integers is closed ?

   (A) $b\ a * b = a/b$. (B) $a * b = a - b$.

   (C) $a * b = ab^2$. (D) $a * b = +\sqrt{ab}$.

3. The order of the subgroup of Z_4 generated by the element 3 is :

   (A) 3. (B) 4.

   (C) 2. (D) 1.

4. The set {1,2,3,4} is an abelian group under the operation ————.

   (A) Addition. (B) Addition modulo 5.

   (C) Multiplication modulo 4. (D) Multiplication modulo 5.

5. Which of the following are true ?

   1 A group may have more than one identity element.

   2 Any two groups of three elements are isomorphic.

   3 Every group of at most three elements is abelian.

   (A) 2 and 3. (B) 1 and 2.

   (C) 1 and 3. (D) All.

6. Let G = C* (non-zero complex numbers). Which of the following are subgroups of G ?

1 All purely imaginary complex numbers under multiplication.

2 All complex numbers with absolute value 1 under multiplication.

   (A) Only 1. (B) Both.

   (C) Only 2. (D) None of these.

7. Which of the following is not true ?

(A) (Z,+) is a proper subgroup off (R,+).

(B) (Q+,.), is a proper subgroup of (R*,.).

(C) (3Z,+) is a proper subgroup of (Z,+).

(D) (Q+,•) is a proper subgroup of (R,+).

8. The Klein 4- group is isomorphic to ————.

(A) $Z_2 \times Z_4$.

(B) $Z_2 \times Z_2$.

(C) Z_4.

(D) None of these.

9. Order of (2, 2) in $Z_4 \times Z_6$ is ————.

(A) 2.

(B) 6.

(C) .4.

(D) 12.

10. The center of an abelian group G is :

(A) $\{e\}$.

(B) G.

(C) A cyclic subgroup.

(D) None of these.

11. If a, b are elements of a group G of order m then order of ab and ba are :

(A) Same.

(B) Equal to m.

(C) Unequal.

(D) None of these.

12. Number of elements in the cyclic subgroup of the group of non-zero complex numbers under multiplication, generated by, $(1+i)/\sqrt{2}$ is :

(A) 4.

(B) 8.

(C) 6.

(D) None of these.

13. Which of the following is not true ?

(A) Every cyclic group of order > 2 has at least two distinct generators.

(B) If every proper subgroup of G is cyclic, then G itself is cyclic.

(C) Every sub-group of a cyclic group is cyclic.

(D) None of these.

14. The number orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 7 & 1 & 4 & 6 & 3 & 5 \end{pmatrix}$ is ————.

(A) 1.
(B) 2.
(C) 3.
(D) 4.

15. The product (1 3 6) (2 4) of two permutation is :

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 6 & 2 & 5 & 1 \end{pmatrix}$.
(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 5 & 4 & 6 \end{pmatrix}$.
(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 6 & 1 & 4 \end{pmatrix}$.
(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 4 & 6 & 1 \end{pmatrix}$.

16. Order of the permutation (4 5) (2 3 7) in S_7 is :

(A) 2.
(B) 3.
(C) 5.
(D) 6.

17. Which of the following is not true ?

(A) Every cycle is a permutation.
(B) A3 is a commutative group.
(C) A5 has 120 elements.
(D) None of these.

18. Index of the subgroup 5Z in (Z,+) is :

(A) 3.
(B) 5.
(C) 7.
(D) Infinite.

19. Suppose G is a group of order 29. Then which of the following is true ?

(A) G is not abelian.
(B) G has no subgroup other than (e) and G.
(C) There is a group H of order 29 which is not isomorphic to G.
(D) G is a subgroup of a group of order 30.

20. The index of the subgroup (3) in Z_{24} :

(A) 3.
(B) 6.
(C) 2.
(D) 4.

D 10231 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. The smallest non abelian group has ——— number of elements.
2. The order of the identity element in any group G is ———.
3. State True or False. "Every abelian group is cyclic".
4. State True or False. "Every group of order 31 is cyclic".
5. Give an example of non-cyclic group with four elements.
6. The total number of subgroups of Z_{12} is ———.
7. What are the orbits of the identity permutation σ of a set A ?
8. How many zero divisors are there for the field Z_7 ?
9. How many units are there for the field Z_7 ?
10. Give an example of integral domain which not a field.
11. State True or False. Z is a sub field of Q.
12. Write the number of generators of the group Z_5 under addition modulo 5.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that left and right cancellation holds in a group G.
14. Let G be a group and suppose that $a * b * c = e \forall a, b, c \in G$. Show that $b * c * a = e$.
15. Prove that a group G has exactly one idempotent element.
16. Can the identity element be a generator of a cyclic group ?
17. Prove that every cyclic group is abelian.

Turn over

18. Consider the group $\mathbb{Z}_{12}$, under the operation addition modulo 12. Find the order of the cyclic subgroup generated by $3 \in \mathbb{Z}_{12}$.
19. Show that the permutation (1, 4, 5, 6) (2, 3, 1, 5) is an even permutation.
20. What is the order of the cycle (1, 4, 5, 7) in S_8 ?
21. Find the partition of the group $\mathbb{Z}_6$, under the operation addition modulo 6, into cosets of the subgroup H = {0, 3}.
22. Consider the rings $\langle \mathbb{Z}, +, \cdot \rangle$ and $\langle 2\mathbb{Z}, +, \cdot \rangle$. Verify whether the map $\phi : \mathbb{Z} \to 2\mathbb{Z}$ defined by $\phi(x) = 2x \forall x \in \mathbb{Z}$ is a ring homomorphism or not.
23. Find the number of generators of the cyclic group of order 8.
24. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.
25. Consider the following two binary structures :
    (a) $\mathbb{Z}$, the set of integers with ordinary addition ; and
    (b) $2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$ the set of even integers with ordinary addition.

    Show that the above two binary structures are isomorphic.
26. Let n be a positive integer. Give an example of a group containing n elements.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Let * be defined by Q^+ by $a * b = \frac{ab}{2}$. Show that $(Q^+, *)$ is an abelian group.
28. Let G be a group. For all $a, b \in G$, prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.
29. Prove that a necessary and sufficient condition that a non-empty subset H of a group G is a subgroup of G is that $a \in H, b \in H \Rightarrow ab^{-1} \in H$.
30. Show that the subgroups of $\mathbb{Z}$ under addition are precisely the groups $n\mathbb{Z}$ under addition for $n \in \mathbb{Z}$.
31. Show that any permutation of a finite set of at least two elements is a product of transpositions.
32. Show that a homomorphism ϕ of a group G is a one-to-one function if and only if Ker $\phi = \{e\}$.
33. Show that cancellation law holds in a ring R if and only if R has no zero divisors.

34. M_2 denotes the ring of all 2 × 2 matrices of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are rational numbers. Is M_2 a field ? Justify your answer.

35. Prove that any integral domain D can be enlarged to (or embedded in) a field F such that every element of F can be expressed as a quotient of two elements of D.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Show that subgroup of a cyclic group is cyclic.

37. (a) Define the term orbit, cycle and transposition with respect to a permutation.

(b) Write the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ as product of disjoint cycles.

(c) Define even and odd permutation. Write (1, 4, 5, 6) (2, 1, 5) as a product of transpositions.

38. Show that in the ring $\mathbb{Z}_n$, the divisors of 0 are precisely those elements that are not relatively prime to n also show that $\mathbb{Z}_p$ is a field.

(2 × 13 = 26 marks)

D 10230–A (Pages : 5) Name.....................................

Reg. No.................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 05 VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 05 VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

1. The length of the vector a with initial point p : (3, 2, 5) and terminal point (5, 1, 3) is :

 (A) 3. (B) 4.

 (C) 5. (D) 6.

2. The straight line through the point (1, 3) in the x, y plane and perpendicular to the straight line $x - 2y + 2 = 0$ is :

 (A) $3x - y = 2$. (B) $x + y = 1$.

 (C) $2x + y = 5$. (D) $2x - y = 5$.

3. The volume of the tetrahedron with co-terminal edges representing the vectors $i + j$, $i - j$ and $2k$ is :

 (A) $\frac{2}{3}$. (B) $\frac{3}{2}$.

 (C) $\frac{3}{5}$ (D) $\frac{2}{5}$.

4. The parametric equations for the line through (– 3, 2, – 3) and (1, – 1, 4) are :

 (A) $x = 1 + 4t, y = - 1 - 3t, z = 4 + 7t$. (B) $x = 2 + 4t, y = - 2 - 3t, z = - 4 + 7t$.

 (C) $x = 3 + 4t, y = 8 - 3t, z = 5 + 7t$. (D) $x = 1 - 4t, y = - 1 + 3t, z = -4 - 7t$.

5. The spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$ is :

 (A) $\Phi = \prod$. (B) $\Phi = \prod/4$.

 (C) $\Phi = \prod/2$. (D) None of these.

6. A particle moves along the curve :

 $x = 3t^2, y = t^2 - 2t, z = t^3$ then the acceleration at $t = 1$ is :

 (A) $6i + 2j + 6k$. (B) $6i + 3k$.

 (C) $6i + 6k$. (D) $6i + 2j + 3k$.

7. The unit tangent vector at a point t to the curve $r = a \cos ti + a \sin t j$:

(A) $-\sin ti - \cos tj$. (B) $-\sin ti + \cos tj$.

(C) $\cos ti - \sin tj$. (D) $-\sin ti + \cos tj$.

8. The domain of the function $f(x, y, z) - xyln(z)$:

(A) Entire Space. (B) $\{(x, y, z) : xyz \neq 0\}$.

(C) Half space $z > 0$. (D) Half space $z < 0$.

9. Which of the following holds for the function $f(x, y) = \dfrac{x+y}{x-y}$?

(A) f is continuous everywhere.

(B) f is continuous nowhere.

(C) f is continuous on $\{(x, y) \in \mathbb{R}^2 : x \neq y\}$.

(D) f is continuous on $\{(x' y) \in \mathbb{R}^2 : x = y\}$.

10. If $f(x, y) = x \sin xy$, then the value of $\dfrac{\partial f}{\partial x}$ at $\left(3, \dfrac{\pi}{6}\right)$ is :

(A) 0. (B) 1.

(C) –1. (D) 2.

11. If $f(x, y) = e^{x+y+1}$ then $\dfrac{\partial f}{\partial y}$ is :

(A) e^{x+1}. (B) e^y.

(C) e^{x+y+1}. (D) NOT.

12. Let $w(p, v, \delta, \gamma, g) = \text{PV} + \dfrac{\gamma\delta^2}{2g}$ then the partial derivative W_δ is given by :

(A) $\dfrac{v}{2g}$.

(B) $pv + \dfrac{vg}{2g}$.

(C) $\dfrac{vg}{g}$.

(D) NOT.

13. The derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction $A = 3i - 4j$ is :

(A) 1.

(B) 0.

(C) – 1.

(D) 2.

14. The unit normal to the surface $x^2y + 2xz = 4$ at the point (2 – 2, 3) is :

(A) $i + j = \text{k}$.

(B) $\dfrac{2}{3}i + 2j + 5k$.

(C) $\dfrac{1}{3}i - j - 7k$.

(D) $\dfrac{1}{3}i - \dfrac{2}{3}j - \dfrac{2}{3}k$.

15. The function $f(x, y) = xy$ has a :

(A) Local maximum.

(B) Local minimum.

(C) Both local maximum and minimum.

(D) No local extreme values.

16. The point $p(x, y, z)$ closest to the origin on the plane $2x + y - z - 5 = 0$ is :

(A) $\left(\dfrac{5}{6}, \dfrac{-5}{6}, \dfrac{5}{6}\right)$.

(B) $\left(\dfrac{5}{3}, \dfrac{5}{6}, \dfrac{5}{3}\right)$.

(C) $\left(\dfrac{5}{3}, \dfrac{5}{6}, \dfrac{-5}{6}\right)$.

(D) $\left(\dfrac{5}{6}, \dfrac{5}{3}, \dfrac{-5}{3}\right)$.

17. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. The points on the Ellipse that lies closest to the origin are :

(A) (1, 0, 0) and (0, 0, 1).

(B) (0, 1, 0) and (0, 0, 1).

(C) (1, 0, 0) and (0, 1, 0).

(D) (1, 0, 0) and (0, 1, 1).

18. Which among the following is the value of $\int_0^1 \int_0^1 xy\,(x - y)\,dxdy$?

(A) 4.

(B) $\frac{2}{3}$.

(C) $\frac{8}{3}$.

(D) $\frac{4}{3}$.

19. The volume enclosed by the co-ordinate planes and the portion of the plane $x + y + z = 1$ in the first octant is :

(A) $1/2$.

(B) $1/3$.

(C) $1/6$.

(D) $1/4$.

20. Which among the following is the value of $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$?

(A) $\frac{1}{8}(26 + \log 27)$.

(B) $\frac{1}{8}(27 - \log 26)$.

(C) $\frac{1}{8}(27 + \log 26)$.

(D) $\frac{1}{8}(26 - \log 27)$.

D 10230 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the domain and range of $z = \sqrt{25 - x^2 - y^2}$.

2. Evaluate $\lim\limits_{(x,y)\to(1,-1)} \dfrac{1+x-y}{2-x+y}$.

3. Define gradient of a scalar function.

4. Compute the divergence of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$.

5. Define solenoidal vector.

6. What do you mean by directional derivative.

7. Write the component test for the differential $M(x, y, z)\,dx + N(x, y, z)\,dy + P(x, y, z)\,dz$ to be exact.

8. Find du if $u = \arcsin \dfrac{x}{y}$.

9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are irrotational vector point functions, then $\nabla.\left(\vec{f} \times \vec{g}\right) = \ldots$.

10. State the normal form of Green's theorem in the plane.

11. Fill in the blanks : If $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then of $\nabla\left(\vec{r}.\vec{a}\right) = \ldots$.

12. State Stoke's theorem.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

Turn over

14. Find the vector normal to the surface $\phi(x,y,z) = xyz$ at $(1, -1,1)$.

15. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (π, π, π) from $\sin(x+y)+\sin(y+z)+\sin(x+z)=0$.

16. Prove that $\nabla\left(r^n\right) = nr^{n-2}\vec{r}$.

17. Compute the average value of the function $f(x,y,z) = xyz$ over the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

18. Evaluate $\int_1^2 \int_3^4 \frac{1}{(x+y)^2}\, dx\, dy$.

19. Leaniarize the function $f(x,y,z) = xy + yz + zx$ at (1, 1, 1).

20. Find the directional derivative of $f(x,y,z) = xy$ at (1, 2).

21. Evaluate $\iint_R (xy)\, dx\, dy$ where R is the positive quadrant of the circle of radius a centred at the origin.

22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix $x = \cos t, y = \sin t, z = t; 0 \le t\, \pi/2$.

23. Test whether $\vec{f} = (yz)\vec{i} + (xz)\vec{j} + (xy)\vec{k}$ is conservative or not.

24. Prove that $div\left(curl\vec{f} = 0\right)$.

25. Verify whether the differential $\left(e^x \cos y + yz\right) dx + \left(xz - e^x \sin y\right) dy\, (xy+z)\, dz$ is exact or not.

26. If S is a closed surface enclosing a volume V then prove that $\iint_S \overline{culf} \cdot \hat{n} dS = 0$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *question.*
Each question carries 7 marks.

27. Using double integrals prove that $\int_0^\infty e^{-\left(x^2\right)} dx = \sqrt{\pi}/2$.

28. Evaluate the line integral $\int_C ydx + xdy$ where C is the boundary of the square $x = 0$, $x = 1$, $y = 0$ and $y = 1$.

29. Find the work done by the force field $\vec{f} = 3xy\vec{i} - 58\vec{j} + 10x\vec{k}$ along the space curve $C: \vec{r} = (t^2+1)\vec{i} + 2t^2\vec{j} + t^3\vec{k}$ where $0 \le t \le 2\pi$.

30. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, – 1, 2).

31. Evaluate the volume bounded by $y = x^2, x = y^2$ and the planes z = 0 and z = 3.

32. Evaluate the area enclosed by the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

33. Find the Local extreme values of $f(x, y) = x^2 + y^2 + xy + 3x - 3y + 4$.

34. Evaluate the line integral $\int_C \vec{f} \cdot \vec{dr}$ where C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0), (1, 1, 0).

35. Show that $\vec{f} = y \sin z\vec{i} + x \sin z\vec{j} + xy \cos z\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** *question.*
Each question carries 13 *marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$ through the surface of the cube cut from the first Octant by the planes $x = y = z = 1$.

(b) Evaluate $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.

37. Verify Stoke's Theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the rectangular region bounded by $x = 0, x = a, y = 0, y = a$.

38. Verify the Tangential form of Green's theorem in the plane for the vector Field $\vec{f} = (x - y)\vec{i} + x\vec{j}$ over the region bounded by the unit circle $x^2 + y^2 = 1$.

(2 × 13 = 26 marks)