C 3557-A (Pages : 4) Name..................................

Reg. No..................................

# FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MEC 4C 04—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

| Time : 15 Minutes | Total No. of Questions : 20 | Maximum : 20 Marks |
| --- | --- | --- |

## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MEC 4C 04—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)

1. Regression analysis is concerned with the study of the dependence of :
   - (A) Explanatory variables on one or more dependent variables.
   - (B) Dependent variable on one or more explanatory variables.
   - (C) Both explanatory and dependent variables on other known variables.
   - (D) Two known variables.

2. Stochastic variables are :
   - (A) Deterministic values.
   - (B) Non-random variables.
   - (C) Imply causation.
   - (D) Have probability distribution.

3. Firm data collected for top 10 companies classified based on profitability for 10 years is an example of :
   - (A) Cross-sectional data.
   - (B) Time series data.
   - (C) Pooled data.
   - (D) Panel data.

4. The data on GDP, unemployment, household expenditure are examples of :
   - (A) Experimental data.
   - (B) Non-experimental data.
   - (C) Cross-section data.
   - (D) Time series data.

5. Variables such as grades in mathematics, results of horse race, degree of satisfaction at a restaurant are examples of :
   - (A) Ratio scale.
   - (B) Interval scale.
   - (C) Ordinal scale.
   - (D) Nominal scale.

6. The sample Regression line is at best an approximation of the true population regression. The statement :
   - (A) Is always true.
   - (B) Is always false.
   - (C) May sometimes be true sometimes false.
   - (D) Non-sense statement.

7. $Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i X + \hat{u}_i$ :
   - (A) Sample regression function.
   - (B) Population regression function.
   - (C) Non-linear regression function.
   - (D) Estimate of regression function.

8. In $Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i X + \hat{u}_i, \hat{u}_i$ represent :

(A) Fixed component. (B) Residual component.

(C) Estimates. (D) Estimators.

9. The regression model includes a random error or disturbance term for a variety of reasons. Which of the following is NOT one of them ?

(A) Individual Y observations are intrinsically random even if they are measured correctly.

(B) Influence of variables other than X (Omitted variables).

(C) Unavailability of measurable data based on theory.

(D) Approximation errors in the calculation of the least squares estimates.

10. In the simple linear regression model, the regression slope :

(A) Indicates by how many percent Y increases, given a one percent increase in X.

(B) When multiplied with the explanatory variable will give you the predicted Y.

(C) Indicates by how many units Y increases, given a one unit increase in X.

(D) Represents the elasticity of Y on X.

11. The statement that -There can be more than one SRF representing a population regression function is :

(A) Always true. (B) Always false.

(C) Sometimes true, sometimes false. (D) Non-sense statement.

12. The population regression function is not directly observable. This is a :

(A) True statement.

(B) False statement.

(C) Mostly true statement depending on the population.

(D) Mostly false statement depending on the observation capacity of researcher.

13. In $Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i X + \hat{u}_i, \hat{u}_i$ gives the differences between :

(A) The actual and estimated Y values.

(B) The actual and estimated X values.

(C) The actual and estimated beta values.

(D) The actual and estimated $u$ values.

14. Under the least square procedure, larger the $\hat{u}_i$ (in absolute terms), the larger the :

    (A) Standard error.

    (B) Regression error.

    (C) Squared sum of residuals.

    (D) Difference between true parameter and estimated parameter.

15. Under normality assumption of $u_i$, the OLS estimator are :

    (A) Minimum variance unbiased.

    (B) Consistent.

    (C) $\hat{\beta}_1$ is normally distributed.

    (D) All the above.

16. Rejecting a true hypothesis results in this type of error :

    (A) Type I error.

    (B) Type II error.

    (C) Structural error.

    (D) Hypothesis error.

17. Accepting a false hypothesis results in this type of error :

    (A) Type I error.

    (B) Type II error.

    (C) Structural error.

    (D) Hypothesis error.

18. The $\alpha$ in a confidence interval given by $Pr\left(\hat{\beta}_2 - \delta \leq \beta_2 \leq \hat{\beta}_2 + \delta\right) = 1 - \alpha$ is known as :

    (A) Confidence coefficient.

    (B) Level of confidence.

    (C) Level of significance.

    (D) Significance coefficient.

19. The $\alpha$ in a confidence interval given by $Pr\left(\hat{\beta}_2 - \delta \leq \beta_2 \leq \hat{\beta}_2 + \delta\right) = 1 - \alpha$ should be :

    (A) $< 0$.

    (B) $> 0$.

    (C) $< 1$.

    (D) $> 0$ and $< 1$.

20. In confidence interval estimation, $\alpha = 5$ %, this means that this interval includes the true $\beta$ with probability of :

    (A) 5 %.

    (B) 50 %.

    (C) 95 %.

    (D) 45 %.

**C 3557** (Pages : 2) Name.......................................

Reg. No...................................

# FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MEC 4C 04—MATHEMATICAL ECONOMICS

Time : Two Hours Maximum : 60 Marks

## Section A

*Answer at least* **eight** *questions.*
*Each question carries 3 marks.*
*All questions can be attended.*
*Overall Ceiling 24.*

1. Distinguish between theoretical econometrics and applied econometrics.

2. What are the goals of Econometrics ?

3. Distinguish between two variable regression and multiple regression analysis.

4. Define Stochastic Disturbance term with stochastic specification of a model.

5. What do you mean by standard error test ?

6. What are the properties of $R^2$ ?

7. Give a Ballentine view of $R^2 = 0$ and $R^2 = 1$.

8. What is Autocorrelation ?

9. What is confidence interval ?

10. Define reciprocal models with its functional form.

11. What do you mean by regression through the origin ?

12. What is forecasting or prediction ?

(8 × 3 = 24 marks)

Turn over

## Section B

*Answer at least* **five** *questions.*
*Each question carries* 5 *marks.*
*All questions can be attended.*
*Overall Ceiling* 25.

13. Explain different measurement scales of variables ?
14. Explain briefly statistical properties of OLS estimators.
15. Distinguish between PRF and SRF.
16. What is Maximum Likelihood Method (ML) ?
17. Explain Type I Error and Type II Error.
18. Why do we adjust $R^2$ ?
19. What do you mean by regression on standardized variables ?

(5 × 5 = 25 marks)

## Section C

*Answer any* **one** *question.*
*The question carries* 11 *marks.*

20. What do you mean by regression ? Explain the method of Ordinary Least Squares.
21. Prove that OLS estimators are the best linear unbiased estimators (BLUE).

(1 × 11 = 11 marks)

**C 3555-A** (Pages : 4) Name..................................

Reg. No..................................

## FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)



| **Time : 15 Minutes** | **Total No. of Questions : 20** | **Maximum : 20 Marks** |
|---|---|---|

## INSTRUCTIONS TO THE CANDIDATE

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## MTS 4B 04—LINEAR ALGEBRA

(Multiple Choice Questions for SDE Candidates)

1. If A and B are square matrices of the same order, then $tr(AB) =$ :

   (A) $tr(A+B)$.
   (B) $tr(A)\,tr(B)$.
   (C) $tr(BA)$.
   (D) $tr(A) + tr(B)$.

2. If A and B are symmetric matrices of same order, then :

   (A) AB is always symmetric.
   (B) AB is never symmetric.
   (C) AB is skew-symmetric.
   (D) AB is symmetric if and only if AB = BA.

3. A matrix that is both symmetric and upper triangular must be a :

   (A) Diagonal matrix.
   (B) Non-diagonal but symmetric.
   (C) Both (A) and (B).
   (D) None of the above.

4. A matrix E is called ———————— if it can be obtained from an identity matrix by performing a single elementary row operation.

   (A) Equivalent matrix.
   (B) Echelon matrix.
   (C) Elementary matrix.
   (D) Row reduced matrix.

5. A homogeneous linear system in $n$ unknowns whose corresponding augmented matrix has a reduced row echelon form with $r$ leading 1's has :

   (A) $n$ free variables.
   (B) $n - r$ free variables.
   (C) $r$ free variables.
   (D) Cannot be determined.

6. If A is an $n \times n$ matrix that is not invertible, then the linear system $Ax = 0$ has :

   (A) Infinitely many solutions.
   (B) Exactly one solution.
   (C) Not possible to find solution.
   (D) Finitely many solutions.

7. If A is an $m \times n$ matrix, then the codomain of the transformation $T_A$ is :

   (A) $R^n$.
   (B) $R^{m+n}$.
   (C) $R^{mn}$.
   (D) $R^m$.

8. If $T_A : R^n \to R^n$ and if $T_A(x) = 0$ for every vector $x$ in $R^n$, then A is :

(A) The $n \times n$ zero matrix.

(B) The $n \times n$ identity matrix.

(C) An elementary matrix.

(D) Cannot be determined.

9. Which of the following is false ?

(A) Every subspace of a vector space is itself a vector space.

(B) Every vector space is a subspace of itself.

(C) The intersection of any *two* subspaces of a vector space V is a subspace of V.

(D) The union of any *two* subspaces of a vector space V is a subspace of V.

10. The polynomials $x-1, (x-1)^2, (x-1)^3$ span $P^3$.

(A) True.

(B) False.

(C) Data not complete.

(D) Span $P^4$.

11. The kernel of a matrix transformation $T_A : R^n \to R^m$ is a subspace of :

(A) $R^n$.

(B) $R^m$.

(C) $R^{n+m}$.

(D) $R^{nm}$.

12. The dimension of zero vector space is :

(A) Not defined.

(B) 1.

(C) 0.

(D) Infinite.

13. Which of the following is true ?

(A) Every linearly independent set of five vectors in $R^5$ is a basis for $R^5$.

(B) Every set of five vectors that spans $R^5$ is a basis for $R^5$.

(C) Every set of vectors that spans $R^5$ contains a basis for $R^5$.

(D) All are true.

14. Which of the following is not a vector space ?

(A) The set of all $2 \times 2$ invertible matrices with the standard matrix addition and scalar multiplication.

(B) The set of all $2 \times 2$ matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with the standard matrix addition and scalar multiplication.

(C) The set of all $2 \times 2$ matrices with real entries with the standard matrix addition and scalar multiplication.

(D) None of these.

15. Let A be any matrix. Then :

(A) $\text{rank}(A) = \text{rank}(A^T)$.

(B) $\text{rank}(A) \neq \text{rank}(A^T)$.

(C) $\text{rank}(A) < \text{rank}(A^T)$.

(D) $\text{rank}(A) > \text{rank}(A^T)$.

16. If W is a subspace of $R^n$, then which of the following statement is false.

(A) $W^{\perp}$ is a subspace of $R^n$.

(B) The only vector common to W and $W^{\perp}$ is 0.

(C) The orthogonal complement of $W^{\perp}$ is W.

(D) None of these.

17. If A is an $m \times n$ matrix, then :

(A) The null space of A and the row space of A are orthogonal complements in $R^n$.

(B) The null space of $A^T$ and the column space of A are orthogonal complements in $R^m$.

(C) Both (A) and (B) are correct.

(D) Neither (A) nor (B) are correct.

18. If A is a $3 \times 5$ matrix, then the rank of $A^T$ is at most :

(A) 4.

(B) 2.

(C) 5.

(D) 3.

19. If A is a $3 \times 5$ matrix, then the nullity of $A^T$ is at most :

(A) 4.

(B) 2.

(C) 5.

(D) 3.

20. The values of $r$ and $s$ for which $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$ has rank 2 ?

(A) $r = 2, s = 2$.

(B) $r = 2, s = 1$.

(C) $r = -2, s = 1$.

(D) Cannot have rank 2.

C 3555 (Pages : 4) Name..........................................

Reg. No..........................................

# FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
## APRIL 2021

Mathematics

MTS 4B 04—LINEAR ALGEBRA

Time : Two Hours and a Half — Maximum : 80 Marks

### Section A (Short Answer Type Questions)

*Answer at least* **ten** *questions.*
*Each question carries* 3 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.



1. Describe different possibilities for solution $(x, y)$ of a system linear equations in the $xy$ plane. What are consistent system ?

2. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ solve the system.

3. Define trace of a square matric. Find the trace of the matrix $A = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$.

4. Show that the standard unit vectors

   $e_1 = (1, 0, \ldots .0), e_2 = (0, 1, 0 \ldots .0), e_3 = (0, 0, 1, 0 \ldots .0) \ldots \ldots e_n = (0, 0 \ldots \ldots 1)$ span $R^n$.

5. Find the co-ordinate vector of $w = (1, 0)$ relative to the basis $s = [\bar{u}_1, \bar{u}_2]$ of $R^2$, where $\bar{u}_1 = (1, -1)$ and $\bar{u}_2 = (1, 1)$.

6. Write two important facts about the vectors in a finite dimensional vector space V.

Turn over

7. Consider the bases $B = [\bar{u}_1, \bar{u}_2]$ and $B' = \left[\bar{u}_1', \bar{u}_2'\right]$ where

$\bar{u}_1 = (1, 0), \bar{u}_2 = (0, 1), \bar{u}_1' = (1, 1), \bar{u}_1' = (2, 1)$. Find the transition matrix $P_{B' \to B}$ from B′ to B.

8. Define row spaces and null spaces an $m \times n$ matrix.

9. If $R = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the row reduced echelon form of a $3 \times 3$ matrix A, then verify the rank-nullity formula.

10. Show that the operator $T : R^2 \to R^2$ that rotates vectors through an angle $\theta$ is one-one.

11. Find the image of the line $y = 4x$ under multiplication by the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

12. Confirm by multiplication that $x$ is an eigen vector of A and find the corresponding eigen value if $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

13. Let A be an $n \times n$ matrix. Define inner product on $R^n$ generated by A. Also write the generating matrix of the weighted Euclidear inner product $< u, v > = w_1u_1v_1 + w_2u_2v_2 + .... + w_nu_nv_n$.

14. If $u, v$ are vectors in a real inner product space V, then show that $\| u + v \| \leq \| u \| + \| v \|$.

15. If A is an $n \times n$ orthogonal matrix, then show that $\| Ax \| = \| x \|$ for all $x$ in $R^n$.

(10 × 3 = 30 marks)

## Section B (Paragraph/Problem Type Questions)

*Answer at least* **five** *questions.*
*Each question carries* 6 *marks.*
*All questions can be attended.*
*Overall Ceiling* 30.

16. Describe Column Row Expansion method for finding the product AB for two matrices A and B. Use this to find the product $AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \\ -3 & 5 & 1 \end{bmatrix}$.

17. If A is an invertible matrix, then show that $A^T$ is also invertible and $(AT)^{-1} = (A^{-1})^T$.

18. Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in $R^3$. Show that $w = (9, 2, 7)$ is a linear combination of $u$ and $v$ and that $w' = (4, -1, 8)$ is not a linear combination of $u$ and $v$.

19. If $s = \{v_1, v_2 \ldots\ldots\ldots v_n\}$ is a basis for a vector space V, then show that every vector $v$ in V can be expressed in form $v = c_1v_1 + c_2v_2 + \ldots. + c_nv_n$ in exactly one way. What are the co-ordinates of $v$ relative to the basis $s$.

20. If A is a matrix with $n$ columns, then define rank of A and show that $\text{rank}(A) + \text{nullity}(A) = n$.

21. Find the standard matrix for the operator $T : R^3 \to R^3$ that first rotates a vector counter clockwise about $z$-axis through an angle $\theta$, then reflects the resulting vector about $yz$ plane and then projects that vector orthogonally onto the $xy$ plane.

22. Define eigen space corresponding to an eigen value $\lambda$ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

23. If $w$ is a sub-space of real inner product space $v$, then show that :

    (a) $w^{\perp}$ is subspace of $v$.

    (b) $w \cap w^{\perp} = \{0\}$.

(5 × 6 = 30 marks)

**Turn over**

## Section C (Essay Type Questions)

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

24. (a) Show that every elementary matrix is invertible and the inverse is also an elementary matrix.

(b) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ using Row operations.

25. (a) Let V be a vector space and $\bar{u}$ a vector in V and K a scalar. Then show that :

(a) $0\,\bar{u} = 0$ ; and

(b) $(-1)\,\bar{u} = -\bar{u}$.

(b) Show that the vectors $v_1 = (1, 2, 1), v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ form a basis for $R^3$.

26. (a) Consider the basis $B = [u_1, u_2]$ and $B' = \left[u_1^1, u_2^1\right]$ for $R^2$ where $u_1 = (2, 2), u_2 = (4, -1)$

$u_1' = (1, 3), u_2' = (-1, -1)$

(i) Find the transition matrix $B'$ to B.

(ii) Find the transition matrix B to $B'$.

(b) Find the reflection of the vector $x = (1, 5)$ about the line through the origin that makes an angle of $\frac{\pi}{6}$ with the $x$-axis.

27. When you can say that a square matrix A is diagonalizable ? If A is an $n \times n$ matrix, show that the following statements are equivalent :

(a) A is diagonalizable ; and

(b) A has $n$ linearly independent eigen vectors.

(2 × 10 = 20 marks)

**C 2218-A** **(Pages : 4)** **Name..................................**

**Reg. No..................................**

**FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION**
**APRIL 2021**

Mathematics

ME 4C 04—MATHEMATICAL ECONOMICS

(Multiple Choice Questions for SDE Candidates)



| **Time : 15 Minutes** | **Total No. of Questions : 20** | **Maximum : 20 Marks** |
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# ME 4C 04—MATHEMATICAL ECONOMICS

## (Multiple Choice Questions for SDE Candidates)

1. The lin log model and log lin model are ———— in parameters.

   (A) Non-linear. (B) Linear.

   (C) Functional. (D) Dependent.

2. ———— is a growth model.

   (A) A linear trend model. (B) Lin log model.

   (C) Log lin model. (D) None of the above.

3. Keynes postulated ———— relationship between income and consumption.

   (A) Negative. (B) Positive.

   (C) Non-linear. (D) Infinite.

4. In the Keynesian linear consumption function $Y = \beta_1 + \beta_2 X$, $\beta_1$ is :

   (A) Slope coefficient. (B) Intercept coefficient.

   (C) Output coefficient. (D) None of the above.

5. The variable appearing on the right side of the equality sign is called :

   (A) Independent variable. (B) Explanatory variable.

   (C) All of the above. (D) None of the above.

6. A mathematical model assumes ———— relationship between variables.

   (A) Inexact. (B) Exact.

   (C) Probable. (D) None of the above.

7. Regression analysis is concerned with :

   (A) Study of the dependence on one variable on the other.

   (B) Predicting the average value.

   (C) Predicting the population mean.

   (D) All of the above.

8. Regression analysis is concerned with ———— relationship among variables.

(A) Statistical.
(B) Functional.
(C) Deterministic.
(D) None of the above.

9. Correlation theory is based on the assumption of :

(A) Randomness of variables.
(B) Conditional mean.
(C) Random errors.
(D) Specification.

10. The law of universal regression was first introduced by :

(A) Irwing Fisher.
(B) Laspayer.
(C) Francis Galton.
(D) Pearson.

11. An expected value is the same as :

(A) Average value.
(B) Standard deviation.
(C) Dispersion.
(D) None of the above.

12. The regression line or curve passes through :

(A) Origin.
(B) Vertical axis.
(C) Horizontal axis.
(D) Conditional means.

13. "The descriptions be kept as simple as possible until proved inadequate" corresponds to :

(A) Occam's razor.
(B) Index numbers.
(C) Regression.
(D) Correlation.

14. The most popular method of constructing sample regression function in the regression analysis is :

(A) Method of OLS.
(B) Generalised squares.
(C) Ordinary regression method.
(D) None of the above.

15. Homoscedasticity means ———— for disturbances.

(A) Equal mean.
(B) Equal variance.
(C) Zero mean.
(D) None of the above.

16. Economic theory makes statements that are mostly :

(A) Quantitative.
(B) Qualitative.
(C) Positive.
(D) None of the above.

17. Heteroscedasticity implies :

(A) Equal spread.
(B) Unequal spread.
(C) Equal mean.
(D) Equal variance.

18. The numerical value of coefficient of determination lies between :

(A) –1 and 1.
(B) 0 and 1.
(C) $-\infty$ to $+\infty$.
(D) $-\infty$ to 1.

19. The rejecting of a true hypothesis is called :

(A) Type I error.
(B) Type II error.
(C) Standard error.
(D) Point estimation.

20. The accepting of a false hypothesis is called :

(A) Type I error.
(B) Type II error.
(C) Standard error.
(D) Point estimation.

C 2218 (Pages : 3) Name..................................

Reg. No...............................

# FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

ME 4C 04—MATHEMATICAL ECONOMICS

Time : Three Hours Maximum : 80 Marks



## Part A

*Answer* **all** *the twelve questions.*
*Each question carries* 1 *mark.*

1. What are the two types of econometrics ?
2. Give an example of an economic model.
3. What is the conditional expectation function or the population regression function ?
4. State Gauss-Markov Theorem.
5. What is the maximum value of the co-efficient of determination ?
6. Define correlation co-efficient.
7. What are the two branches of classical theory of statistical inference ?
8. What is the mean and variance of a standard normal variable ?
9. Define 'confidence interval'.
10. What is type I error ?
11. Give an example of a regression model that not linear in the variables.
12. What is Logarithmic Reciprocal Model ?

($12 \times 1 = 12$ marks)

## Part B

*Answer any* **six** *questions in two* **or** *three sentences.*
*Each question carries* 3 *marks.*

13. Write a short note on Keynesian consumption function.
14. What is the difference between the population and sample regression functions ? Illustrate with an example.

Turn over

15. Given $E(u_i|X_i) = 0$, show that $E(Y_i|X_i) = \beta_2 + \beta_2 X_i$.

16. Show that the mean value of the estimated $Y = \hat{Y}_i$ is equal to the mean value of the actual Y in a sample : $\{(x_i, y_i) : i = 1, 2, ...., n\}$.

17. What are the classical normal linear regression model assumption for $\mu_i$.

18. Under the normality assumption find the 100 (1 – α) % confidence interval for regression co-efficient $\beta_2$

19. Write a short note on Testing of hypothesis.

20. What are the three main features of reciprocal models ?

21. Find the elasticity of the log linear regression model $\ln Y = \beta_1 + \beta_2 \ln X$.

(6 × 3 = 18 marks)

**Part C**

*Answer any* **six** *questions from the following.*
*Each question carries* 5 *marks.*

22. Write any *five* properties of coefficient of correlation.

23. Show that the least-squares estimator $\hat{\beta}_2$ is linear.

24. Show that $\hat{\sigma}^2$ is an unbiased estimator of true $\sigma^2$.

25. What are the 5 important properties of OLS Estimators under the Normality Assumption on $u_i$.

26. Briefly discuss about maximum likelihood estimation of Two variable regression model.

27. Find the Confidence Intervals for Regression Co-efficients $\beta_1$.

28. Consider the following regression model : $1/Y_i = \beta_1 + \beta_2 (1/X_i) + u_i$.

(a) Is this a linear regression model ? Why ? Why not ?

(b) How would you estimate this model ?

(c) What is the behavior of Y as X tends to infinity ?

(d) Can you give an example where such a model may be appropriate ?

29. How to measure the growth rate using the LogLin model ?

30. What is Log Linear regression model ? How to measure elasticity using this model ?

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions from the following.*

*Each question carries* 10 *marks.*

31. Discuss various steps involved in the traditional econometric methodology.

32. Write the 10 Assumptions made in the classical linear regression model.

33. In the following table, you are given the ranks of 10 students in midterm and final examinations in mathematics. Compute Spearmans co-efficient of rank correlation and interpret it.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Midterm | 1 | 3 | 7 | 10 | 9 | 5 | 4 | 8 | 2 | 6 |
| Final | 3 | 2 | 8 | 7 | 9 | 6 | 5 | 10 | 1 | 4 |

34. Why do we employ the normality assumption on $\mu_i$.

(2 × 10 = 20 marks)

C 2217 (Pages : 4) Name.......................................

Reg. No..................................

# FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

## Part A

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. What do you mean by a non-linear differential equation ?
2. What are the steps for finding general solution of a non-homogeneous equation $y'' + ay' + by = r(x)$.
3. Find Wronskian of $y_1(x) = e^{-2x}$ and $y_2(x) = e^{-3x}$.
4. What is $L[1]$?
5. Define periodic function.
6. What is unit step function ?
7. State Convolution theorem.
8. Define and give an example of an even function.
9. Give one dimensional wave equation.
10. Write the formula for Runge Kutta method.
11. Give formula for Euler method.
12. Give a formula for an error for Simpson's rule.

(12 × 1 = 12 marks)

## Part B

*Answer any* **nine** *questions.*
*Each question carries 2 marks.*

13. Find the particular integral for $y'' - 4y' + 3y = 10e^{-2x}$.

14. Solve $\left(D^2 - 2D + 3\right)y = x^3 + \sin x$.

15. Find $W\left[e^{\lambda_1 x}, e^{\lambda_2 x}\right]$.

16. If $L^{-1}(f(s)) = F(t)$ then show that $L^{-1}(f(s-a)) = e^{at} F(t)$.

17. Show that the Laplace transform is a linear operation.

18. Find $L\left[t^2 \cos t\right]$.

19. Using convolution property, find $L^{-1}\left[\dfrac{1}{s^2(s-a)}\right]$.

20. Find the Fourier series of $f(x) = x^2$, when $-1 < x < 1$ with period 2.

21. Show that $u = \cos 4t \sin 2x$ is a solution of the wave equation.

22. Apply Picard's iteration upto 3 steps to solve $y' = 1 + y^2$ and $y(0) = 1$.

23. Compute $\int_0^1 x^2\, dx$ by the rectangular rule with $h = 0.5$.

24. Solve $\int_1^2 \frac{1}{x}\, dx$ by Trapezoidal rule with $n = 4$ and compare the estimate with the exact value of the integral.

(9 × 2 = 18 marks)

## Part C

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Solve $x^2 y'' + 7xy' + 13y = 0$.

26. Solve the non-homogeneous equation $y'' - 4y' + 3y = 10e^{-2x}$.

27. Obtain the Fourier cosine series representation of $f(x) = e^x, x \in [0, \pi]$.

28. Find the inverse transform of $\dfrac{s^3 - 4s^2 + 4}{s^2(s^2 - 3s + 2)}$.

29. Solve $u_x + u_y = 2(x + y)u$.

30. Express the function $f(x) = x^2$, when $-1 < x < 1$ as a Fourier series with period 2.

31. Solve the integral equation $y = 1 - \int_0^t (t - \tau) y(\tau) d\tau$.

32. Find an approximate value of $\log_e 5$ by calculating $\int_0^5 \dfrac{dx}{4x + 5}$ by Simpson's rule of integration.

33. Solve by Picard's method $y' - xy = 1$, given $y = 0$ when $x = 2$. Also find $y(2.05)$ correct to four places of decimal.

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. (a) Solve $x^2y'' - 4xy' + 6y = 21x^{-4}$.

(b) Solve $(D^2 - 2D + 1)y = 3x^{3/2}e^x$.

35. Find the solution of the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

corresponding to the triangular initial deflection

$$f(x) = \begin{cases} \frac{2k}{l} x, & \text{when } 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l - x), & \text{when } \frac{l}{2} < x < l \end{cases}$$

and the initial velocity zero.

36. Find the Fourier series of $f(x) = \begin{cases} 2, & -2 \le x < 0 \\ x, & 0 \le x < 2 \end{cases}$ in $(-2, 2)$.

(2 × 10 = 20 marks)

**C 2216-A** **(Pages : 4)** Name..................................

Reg. No..................................

# FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
## APRIL 2021

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

| Time : 15 Minutes | Total No. of Questions : 20 | Maximum : 20 Marks |
|---|---|---|



## INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

# MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)



1. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ is :

   (A) 0. (B) 1.

   (C) 2. (D) 3.

2. The rank of the matrix $\begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ is :

   (A) 3. (B) $4 \times 3$.

   (C) 2. (D) 1.

3. Rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$ is :

   (A) 1. (B) 2.

   (C) 3. (D) 4.

4. The points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear if and only if the rank of the matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ is :

   (A) $< 3$. (B) $\leq 3$.

   (C) $> 3$. (D) $\geq 3$.

5. If a matrix A has a non-zero mirror of order $r$, then :

   (A) $\rho(A) = r$. (B) $\rho(A) \geq r$.

   (C) $\rho(A) < r$. (D) $\rho(A) \leq r$.

6. Which of the following is false :

   (A) $\rho(A+B) \leq \rho(A)+\rho(B)$.

   (B) $\rho(A') = \rho(A)$.

   (C) $\rho(A+B) = \rho(A)+\rho(B)-4$, if A and B are matrices of rank $z$.

   (D) $\rho(A-B) \leq \rho(A)\rho(B)$.

7. Rank $(AA') =$ ————————.

   (A) Rank A. (B) Rank $A'$.

   (C) 1. (D) None.

8. Rank $(AA^{\theta})$ = ————————.

   (A) Rank $A^{\theta}$. (B) Rank A.

   (C) Rank $A'$. (D) None.

9. The system AX = 0 in $n$ unknowns has a non-trivial solution if :

   (A) $\rho(A) > n$. (B) $\rho(A) = n$.

   (C) $\rho(A) < n$. (D) None of these.

10. A system of M homogeneous linear equations AX = 0 in $n$ unknown has only trivial solution if :

    (A) $m = n$. (B) $m \neq n$.

    (C) $\rho(A) = m$. (D) $\rho(A) = n$.

11. The system of equations $\begin{array}{l} x+2y+z=9 \\ 2x+y+3z=7 \end{array}$ can be expressed as :

    (A) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. (B) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

    (C) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$. (D) None.

12. If A is a square matrix of order $n$ and $\lambda$ is a scalar, then the characteristic polynomial of A is obtained by expanding the determinant.

(A) $|\lambda A|$.
(B) $\lambda|A|$.
(C) $|\lambda A - I_n|$.
(D) $|A - \lambda I_n|$.

13. The characteristic roots of Skew-Hermitian matrix are either :

(A) Real or zero.
(B) Real or non-zero.
(C) Pure imaginary or zero.
(D) Pure imaginary non-zero.

14. The matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation :

(A) $A^2 + 5A + 7I = 0$.
(B) $A^2 + 5A - 7I = 0$.
(C) $A^2 - 5A - 7I = 0$.
(D) $A^2 - 5A + 7I = 0$.

15. The product of all the characteristic roots of a square matrix A is equal to :

(A) 0.
(B) 1.
(C) $|A|$.
(D) $\frac{1}{|A|}$.

16. If eigen value of matrix A is $\lambda$, then eigen value of $P^{-1}AP$ is :

(A) 1.
(B) $\lambda$.
(C) $\frac{1}{\lambda}$.
(D) 0.

17. A polynomial equation in $x$ of degree $n$ always have :

(A) $n$ distinct roots.
(B) $n$ real roots.
(C) $n$ complex roots.
(D) None.

18. A zero of the polynomial $x^3 + 2x - i$ equals :

(A) $-i$.
(B) 1.
(C) $1 - i$.
(D) None.

19. If $\alpha, \beta, \gamma$ are the roots of $x^3 + px^2 + qx + r = 0$ then $\alpha\beta + \beta\gamma + \gamma\alpha$ equals :

(A) $\frac{-p}{q}$.
(B) $-p$.
(C) $q$.
(D) $-q$.

20. A polynomial equation whose roots are 3 times those of the equation $2x^3 - 5x^2 + 7 = 0$ is :

(A) $3x^3 - 15x^2 + 21 = 0$.
(B) $2x^3 - 15x^2 + 189 = 0$.
(C) $2x^3 + 15x^2 - 189 = 0$.
(D) None.

C 2216

(Pages : 4)

Name..............................

Reg. No..............................

# FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
## APRIL 2021

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours | Maximum : 80 Marks



### Part A (Objective Type)

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. If $\alpha, \beta, \gamma$ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\alpha^2, \beta^2, \gamma^2$.

2. State Descarte's rule of signs.

3. Find a cubic equation, two of whose roots are given by $1, 3 + 2i$.

4. What do you mean by reciprocal equation of first type ? Give example.

5. What is the rank of the identity matrix of order 101 ?

6. If $A = [a_{i,j}]$ is an $m \times n$ matrix and $a_{i,j} = 7$ for all $i, j$ then rank of A is ——————.

7. A system of $m$ homogeneous linear equations in $n$ unknowns has only trivial solution if ——————.

8. For what values of $a$ the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.

9. If the number of variables in a non-homogeneous system AX = B is $n$ then the system possesses a unique solution if ——————.

10. Find the parametric equation of a line through $P(1, 1, 1)$ and parallel to the $z$-axis.

11. Find the unit tangent vector of the helix $r(t) = (\cos t + t \sin t)\, i + (\sin t - t \cos t\ j)\, j + tk, t > 0$.

12. Write equations relating spherical and cylindrical co-ordinates.

$(12 \times 1 = 12$ marks$)$

Turn over

## Part B (Short Answer Type)

*Answer any* **nine** *questions.*
*Each question carries 2 marks.*

13. Solve $6x^3 - 11x^2 - 3x + 2 = 0$. Given that the roots are in harmonic progression.

14. Find the equation whose roots are the roots of $x^3 + 3x^2 - 2x - 4 = 0$ increased by 5.

15. If $\alpha, \beta, \gamma$ are the roots of $x^3 + qx + r = 0$. Find the equation whose roots are $(\beta - \alpha)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

16. If $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$. Find $A^{-1}$.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If $\alpha$ is an eigen value of a non-singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order $n$ is equal to the determinant of the matrix.

20. Find the value of $a$ for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$.

21. Find the velocity and acceleration vectors of $r(t) = (t+1)i + (t^2 - 1)j + 2tk$ at $t = 1$.

22. Find the rectangular co-ordinates of the centre of the sphere $r^2 + z^2 = 4r\cos\theta + 6r\sin\theta + 2z$.

23. Evaluate $\int_0^{\pi} (\cos ti + j - 2tk)\, dt$.

24. Find the principal unit normal for the circular motion $r(t) = (\cos 2t)i + (\sin 2t)j$.

(9 × 2 = 18 marks)

## Part C (Short Essay)

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. If $\alpha, \beta, \gamma$ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by Cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of the transpose of a matrix is equal to the rank of the same matrix.

29. Find the rank of $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{pmatrix}$.

30. Using matrix method solve :

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2.$$

31. Find the point in which the line $x = 1 + 2t, y = 1 + 5t, z = 3t$ intersects the plane $x + y + z = 2$.

32. Find the distance from the point $S(2, 1, 3)$ to the line $x = 2 + 2t, y = -1 + 6t, z = 3$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Solve the equation $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$.

35. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$ and hence evaluate $A^{-1}$.

36. Find the binormal vector and torsion for the space curve $r(t) = \left(\frac{t^3}{3}\right) i + \left(\frac{t^2}{2}\right)$.

(2 × 10 = 20 marks)