C 23382 (Pages : 2) Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE [REGULAR/SUPPLEMENTARY] EXAMINATION, APRIL 2022

(CBCSS)

Statistics

MST 2C 08—TESTING OF STATISTICAL HYPOTHESIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Weight 2 for each question.

1. What do you mean by *p*-value ? Give the merit of *p*-value approach over significance level approach.
2. Define MLR property. Mention an example where this property lacks.
3. Define UMPU tests. Give an example of a UMPU test.
4. Define Neymann structure tests.
5. Define empirical distribution function. Show that it is an unbiased estimator of the CDF.
6. Mention the significance of OC curves in testing of hypotheses.
7. What are the demerits of SPRT ?

($4 \times 2 = 8$ weightage)

Turn over

Part B

Answer any **four** *questions.*
Weight 3 for each question.

8. A random sample of size 2 is taken from a point binomial distribution to test $H_0 : p = 0.6$ against $H_1 : p = 0.4$. Obtain the best test of size $\alpha = 0.1$.

9. Obtain the UMP test of size α based on a random sample of size n from $U(0, \theta)$ for testing :

   $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

10. Explain the construction of LMP tests.

11. Establish the relation between similar region tests and Neymann structure tests.

12. Obtain the likelihood ratio test for testing $H_0 : \sigma^2 = \sigma^2{}_0$ against $H_1 : \sigma^2 \neq \sigma^2{}_0$ in $N\left(\mu, \sigma^2\right)$ where both parameters are unknown.

13. Compare chi-square and Kolmogorov-Smirnov goodness of fit tests.

14. Let X follows exponential distribution with mean λ. Find SPRT of strength (α, β) for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ $(\lambda_1, < \lambda_0)$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Weight 5 for each question.

15. State and prove Neymann Pearson Lemma. Show that Neymann Pearson most powerful test is always unbiased.

16. Define likelihood ratio test criterion. Obtain its asymptotic distribution stating clearly the assumptions.

17. Describe Mann-Whitney U-test. Establish its connection with Wilcoxon statistic.

18. Derive the approximate expressions for OC and ASN functions of an SPRT of strength (α, β).

(2 × 5 = 10 weightage)

C 23381 **(Pages : 2)** **Name..................................**

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Statistics

MST 2C 07—SAMPLING THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
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Part A

Answer any **four** *questions.*
Each question carries weightage 2.

1. Explain the terms : (a) Bias ; and (b) Mean Square Error.
2. What do you mean by census method and method of sampling ?
3. What are the advantages of stratified sampling when compared to SRS ?
4. What is regression estimator of population mean ? Is it unbiased ?
5. Discuss the use of auxiliary information in sample surveys.
6. Define single stage cluster sampling.
7. Describe Lahiri's method of selection under PPS sampling.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries weightage 3.

8. Show that the sample mean is the BLUE of the population mean in SRS WOR procedure.

9. With usual notations show that ratio estimator is better than the SRSWOR when

$$\rho > (1/2)\left[C_x / C_y\right].$$

10. Describe various factors of non sampling errors.

11. Give an unbiased estimator of the population mean based on a stratified random sample. Obtain its variance.

12. Explain Hartly - Ross unbiased ratio type estimator.

13. In a single stage cluster sampling with clusters of unequal size, obtain the three different estimators of the population mean. Examine whether they are unbiased. Obtain the variance of any one of them.

14. Define Horwitz - Thompson estimator under PPS WOR scheme, for the population total. Is it unbiased ? Obtain the expression for its variance.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

15. Define simple random sampling. Suggest an unbiased estimator of the population mean in SRSWOR. Derive the variance of the estimator.

16. With usual notations, prove that, in stratified sampling :

$$V_{opt} \leq V_{prop} \leq V_{ran}$$

17. Define ratio estimators. Is it unbiased ? Derive the first order expression for the bias.

18. Define multi-stage sampling ? In two stage cluster sampling with clusters of equal size, suggest an unbiased estimator of the population mean and derive its variance.

(2 × 5 = 10 weightage)

C 23380 (Pages : 3) Name...

Reg. No....................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Statistics

MST 2C 06—ESTIMATION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

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Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. Define minimal sufficient statistic.
2. State Basu's Theorem.
3. Prove or disprove : Unbiased estimators are always consistent.
4. Give an example to show that Maximum Likelihood Estimator is not always unique.
5. Define consistency of an estimator.
6. Let X be an observation form $B(l, p)$, then show that based on X, unbiased estimator for l/p does not exist.
7. Explain the method of minimum Chi-square for estimation of population parameter.

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Prove that $\bar{X}$, the mean of a random sample of size n from a normal population with mean μ and known standard deviation σ, is an efficient estimator of μ .

9. Derive necessary and sufficient condition for an unbiased estimator to be UMVUE.

10. Given $X_1, X_2, \ldots, X_n$ are n independent observations drawn from the Poisson Distribution with parameter λ. Then obtain the m.l.e of $e^{-\lambda}$.

11. Show that $\dfrac{\frac{1}{2}\left(\sum X_i^2\right)^{1/2} \Gamma\frac{n}{2}}{\Gamma\frac{n+1}{2}}$ is unbiased for θ where $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N\left(0, \theta^2\right)$.

12. Explain shortest length confidence interval. Let $X_1, X_2, \ldots, X_n$ be a sample from $U(0, \theta)$, find shortest length confidence interval for θ.

13. If n independent observations $X_1, X_2, \ldots, X_n$ are drawn from the Poisson distribution with parameter λ, show that the estimator $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} i X_i$ is consistent for λ.

14. Prove that under regularity conditions m.l.e of $\theta, \hat{\theta} \sim AN\left(\pi\theta, \frac{1}{nI(\theta)}\right)$, where $I(\theta)$ is the fisher information.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

15. a) Define minimum variance bound (MVB) estimator. State and prove Cramer Rao inequality.

    b) Find the lower bound for the variance of unbiased estimator for σ based on a random sample of size n from $N(\mu, \sigma^2)$.

16. State and prove Rao-Blackwell theorem.

17. (a) Explain the term Risk. What is meant by minimax estimator ? Show that Bayes estimator with constant risk is minimax.

    (b) If n independent observations $X_1, X_2, ..., X_n$ are drawn from the p.d.f. $f(x, \theta) = e^{(\theta - x)}, x > \theta$.

    Examine whether $X_{(1)}$ is unbiased and consistent for θ.

18. (a) State and prove Fisher-Neyman Factorization theorem.

    (b) Let $X_1, X_2, ..., X_n$ be a random sample from $U(-\theta, \theta)$. Obtain sufficient statistic for θ.

(2 × 5 = 10 weightage)

C 23379 (Pages : 2) Name.......................................

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Statistics

MST 2C 05—DESIGN AND ANALYSIS OF EXPERIMENTS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
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Part A

Answer any **four** *questions.*
Each answer carries 2 *weightage.*

1. Give the ANOVA model of a Latin square design with the assumptions.
2. Explain how a missing value is estimated in a completely randomized design.
3. Define a balanced incomplete block design, and give an example.
4. What is meant by inter-block and intra-block analysis in the context of incomplete block designs?
5. Define confounding in factorial experiments, and give its use.
6. Explain the structure of the treatments combinations in a 2^2 factorial experiment.
7. Describe the objective of response surface methodology.

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each answer carries 3 weightage.

8. Explain the three principles of experimentation.
9. Explain the difference between ANOVA and ANCOVA procedures in comparing treatments.
10. Show that for a balanced incomplete block design, $b \geq v$.
11. Give an example of a lattice design.
12. Explain the nature and use of fractional factorial designs.
13. Explain the specialty of split-plot designs, and give an example.
14. Explain the idea of orthogonality in the context of first-order response surface designs.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

15. Give the ANOVA of a randomized block design with unequal number of observations per cell.
16. Give the intra-block analysis of a balanced incomplete block design.
17. A 2^5 experiment is wished to be conducted using blocks of size 8. Give the sets of treatment combinations to be used in various blocks if BCD and ADE are to be confounded.
18. Explain in detail any one method of constructing a second order rotatable designs.

(2 × 5 = 10 weightage)