D 13152 (Pages : 3) Name..............................

Reg. No..............................

FIRST SEMESTER M.A./M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020/2021

(CBCSS)

Mathematics with Data Science

MTD 1C 05—STATISTICAL INFERENCE AND COMPUTING USING R

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Give an example of an estimator which is sufficient, consistent but not unbiased.
2. Derive large sample confidence interval for population proportion.
3. Explain Bayes estimation.
4. Define type I error, type II error, size and power of a test.
5. Narrate various advantages and disadvantages of a non-parametric test.
6. What do you mean by p-value ?
7. Write R command to select a random sample of size 5 from the following data :

   1, 3, 5, 6, 7, 8, 10, 11, 12, 141, 3, 5, 6, 7, 8, 10, 11, 12, 14.
8. Explain with examples the concept of objects in R.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions, choosing* **two** *questions from each unit.*
Each question has weightage 2.

Unit-1

9. What do you mean by UMVUE ? Obtain UMVUE of $e^{-\lambda}$ in Poisson(λ) based on a sample of size n.

10. State and prove necessary and sufficient condition for the attainment of Cramer-Rao lower bound. Examine such an estimator exist for μ in the case $N(\mu,1)$ population.

11. Obtain the moment estimator of m and p based on a sample of size n from $X \sim G(m, p)$.

Unit-2

12. Obtain the MP test for testing $\mu = \mu_0$ against $\mu = \mu_1 (\mu_1 > \mu_0)$ when $\sigma^2 = 1$ in normal population, $N(\mu, \sigma^2)$.

13. Following are the yields of maize in quintal/hectare recorded from an experiment and arranged in ascending order

    15.4, 16.4, 17.3, 18.2, 19.2, 20.9, 22.7, 23.6, 24. Test $H_0 : M = 20$ Vs $H_1 : M > 20$ at $\alpha = 0.05$ using Wilcoxon signed rank test, where M is the population median.

14. Explain Chi-square test for goodness of fit and compare it with K-S test.

Unit-3

15. Write a R program to create a 4 × 4 matrix taking a given vector of numbers as input and define the column and row names. Display the matrix. And also write a R program to access the element at 3rd column and 2nd row, only the 3rd row and only the 4th column of a given matrix.

16. Explain any six built-in functions in R.

17. Explain types of loop in R programming.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. Given a random sample of n observation on X with $X \sim G(\alpha, p)$, where p is known. If prior distribution of α is $G(\theta, \delta)$, find the posterior distribution of α and also find Bayes estimator of α under squared error loss function.

19. Show that MLE of a parameter is asymptotically normal under some regularity conditions.

20. State and prove Neyman-Pearson lemma.

21. Write a note on data input methods in R.

$(2 \times 5 = 10$ weightage)

D 13151 **(Pages : 3)** Name.....................................

Reg. No.................................

FIRST SEMESTER M.A./M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020/2021

(CBCSS)

Mathematics with Data Science

MTD 1C 04—MEASURE AND PROBABILITY

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Show that every interval is a Borel set.
2. Show that rational equivalent* defines an equivalence relation on any set.
3. Let A and B be any sets. Show that :

$$\chi A \cap B = \chi A \cdot \chi B$$
$$\chi A \cup B = \chi A + \chi B - \chi Z \cdot \chi B$$
$$\chi A^{C} = 1 - \chi A.$$

4. Let E have measure zero. Show that if f is a bounded function on E, then f is measurable and $\int_E f = 0$.
5. State Monotone convergence theorem, also state Lebesgue Dominated convergence theorem.

Turn over

6. Let f be a bounded function on $[a, b]$ whose set of discontinuity has measure zero. Show that f is measurable.

7. Show that every function X on Ω is measurable with respect to the power set of Ω.

8. Show that $P(A \Delta B) = P(AB^c \cup BA^c) = P(A) + P(B) - 2P(AB)$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each units.*
Each question carries weightage 2.

Unit 1

9. Show that the union of a countable collection of measurable sets is measurable.

10. Let the function f be defined on a measurable set E. Then show that f is measurable if and only if for each open set θ, the inverse image of θ under $f, f^{-1}(\theta) = \{x \in E \mid f(x)\}$ is measurable.

11. Under the assumptions of Egoroffs theorem, show that for each $\eta > 0$ and $\delta > 0$, there is a measurable subset A of E and an index N for which $|f_n - f| < \eta$ on A for all $n \geq N$ and $m(E \sim A) < \delta$

Unit 2

12. Let f be a bounded function defined on the closed, bounded interval $[a, b]$. Show that if f is Reimann integrable over $[a, b]$, then it is Lebesgue integrable over $[a, b]$ and the two integrals are equal.

13. Let f be integrable over E and $\{E_n\}_{n=1}^{\infty}$ a disjoint countable collection of measurable subsets of E whose union is E. Then show that $\int_E f = \sum_{n=1}^{\infty} \int_{E_n} f$.

14. Assume E has finite measure. Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise a.e on E to f and f is finite a.e. on E. Then show that $\{f_n\} \to f$ in measure on E.

Unit 3

15. Show that :

    (i) If ε is a field or a σ- field of subsets of Ω', then $X^{-1}(\varepsilon)$ is a field (or a σ-field) of subsets of Ω.

    (ii) Inverse image of the minimal σ- field over any class ε is the minimal σ-field over $X^{-1}(\varepsilon)$, i.e. $\sigma\{X^{-1}(\varepsilon)\} = X^{-1}\{\sigma(\varepsilon)\}$.

16. Show that the distribution function F_x of r.v. X is non-decreasing, continuous on the right with $F_X(-\infty) = 0$ and $F_X(+\infty) = 1$. Conversely, show that every function F, with the above properties is the d.f of a r.v. on some probability space.

17. If Z is a complex r.v., then show that $|EZ| \leq E|Z|$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

18. Prove that Lebesgue measure processes the following continuity properties :

(i) If $\{A_k\}_{k=1}^{\infty}$ is an ascending collection of measurable sets, then :

$$m\left(\cup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(B_k).$$

(ii) If $\{B_k\}_{k=1}^{\infty}$ is a descending collection of measurable sets and $m(B_1) < \infty$,

then $m\left(\cap_{k=1}^{\infty} B_k\right) = \lim_{k\to\infty} m(B_k)$.

19. (a) Let f_n be a sequence of measurable functions on E that converges pointwise a.e. on E to the function f. Then show that f is measurable.

(b) State and prove Lusin's theorem.

20. (a) State and prove Fatou's lemma.

(b) Let f be a measurable function on E. If f is integrable over E, then show that for each $\in > 0$, there is a $\delta > 0$ for which

if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \varepsilon$ (1)

Conversely show that in the case $m(E) < \infty$, if for each $\in > 0$, there is a $\delta > 0$ for which the above condition (1) holds, then f is integrable over E.

21. (a) State and prove Kolmogorov 0 - 1 law.

(b) Sate and prove Borel a.s. criterion.

(2 × 5= weightage)

D 13150 **(Pages : 3)** Name.....................................

Reg. No..................................

FIRST SEMESTER M.A./M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020/2021

(CBCSS)

Mathematics with Data Science

MTD 1C 03—REAL ANALYSIS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Define a countable set. Give one example
2. Define uniformly continuous function between metric spaces. Give one example.
3. If f is a real valued function defined on a metric space X, then what do you mean by the statement f has a local maximum at a point p of X.
4. If f is differentiable in (a, b) and $f'(x) \geq 0$, for all x in (a, b), then show that f is monotonically increasing in (a, b).
5. What do you mean by a Riemann integrable function ?
6. Give an example of a function which is not Riemann Integrable.
7. Give an example of a sequence of uniformly convergent functions.
8. State Stone Weierstrass theorem.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer **six** *questions choosing any 2 from each unit.*
Each question carries weightage 2.

Unit I

9. Prove that an infinite subset of a countable set is countable.

10. Prove that $d(x,y)=\dfrac{|x-y|}{(1+|x-y|)}$ is a metric in R

11. If f is a function from metric space X to metric space Y, then prove that f is continuous on X iff $f^{-1}(V)$ is open for all V open in Y.

Unit II

12. Find the value of $\lim_{x\to 0}\left(\dfrac{\sin x}{x^2}\right)$.

13. Show that the L'Hospital's rule fails in case of complex valued functions with a suitable example.

14. If P^* is a refinement of a partition P of $[a,b]$, then prove that $L(P,f,\alpha) \le L(P^*,f,\alpha)$.

Unit III

15. If $f(x) = c$, a constant on $[a, b]$, show that f is Riemann integrable on $[a, b]$.

16. Show by an example that the order of the limit in a double sequence cannot be interchanged in general.

17. 1f X is a metric space and C(X) is the set of complex valued bounded continuous functions on X. Show that there exists a norm on C(X), which makes it complete.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

18. a) Let A be the set of all sequences whose elements are digits 0 and 1. Prove that A is uncountable.

(2 weightage)

b) X is an infinite set- with discrete metric. Which subsets of X are open ? Which subsets of X are closed ? Which subsets of X are compact ? Explain.

(3 weightage)

19. Prove that every non-empty perfect set in $\mathbb{R}^K$ is uncountable.

20. a) If f is monotonic on $[a, b]$ and monotonically increasing function α is continuous on $[a, b]$ then prove that $f \in R(\alpha)$ on $[a, b]$.

(2 weightage)

b) If f is bounded on $[a, b]$, f has only finite points of discontinuity on $[a, b]$. If the monotonically increasing function α is continuous at every point at which f is discontinuous, then prove that $f \in R(\alpha)$ on $[a, b]$.

(3 weightage)

21. a) If K is a compact metric space, if $\{f_n\} \in C(K)$, for $n = 1, 2, 3, \ldots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

(3 weightage)

b) Prove that the limit of a sequence of uniformly convergent continuous function is continuous.

(2 weightage)

[2 × 5 = 10 weightage]

D 13149 **(Pages : 4)** **Name.....................................**

Reg. No..................................

FIRST SEMESTER M.A./M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020/2021

(CBCSS)

Mathematics with Data Science

MTD 1C 02—LINEAR ALGEBRA

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer **all** *questions.*
Each question has weightage 1.

1. Is $W = \{(a_1, \ldots, a_n) / a_2 = a_1^2\}$ a subspace of $\mathbb{R}^n$.
2. Let F be afield and let $(x_1, \ldots, x_n)$ be a vector in F^n. Find $[\alpha]_B$, where B is the standard ordered basis for F^n.
3. Prove that the map $f : R^2 \to R^3$ defined by $f(x, y) = (x+1, 2y, x+y)$ is not linear.
4. Let V be a vector space and let V^* be the collection of all linear functionals on V. Show that $\dim V^* = \dim V$.
5. Show that similar matrices have the same characteristic polynomial.
6. Let T be a linear operator on V and let U be any linear operator on V which commutes with T, i.e., TU = UT. Let W be the range of U and let N be the null space of U. Show that both W and N are invariant under T.

7. Define inner product on a vector space V.

8. Verify that the standard inner product on F^n is an inner product.

(8 × 1 = 8 weightage)

Section B

Answer any **six** *questions, choosing two questions from each unit.*
Each question has weightage 2.

Unit 1

9. Let W_1 and W_2 be subspaces of a vector space V such that the set -theoretic union of W_1 and W_2 is also a subspace. Prove that one of the subspaces W_1 or W_2 is contained in the other.

10. Let $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1,0,-1), \alpha_2 = (1,1,1), \alpha_3 = (1,0,0)$. What are the co-ordinates of the vector (a, b, c) in the ordered basis B.

11. Let V, W and Z be vector spaces over the field F. Let T be a linear transformation from V into W and U be a linear transformation from W into Z. Then show that the composition function (UT) defined by $(UT)(\alpha) = U(T(\alpha))$ is a linear transformation from V into Z.

Unit 2

12. Let $T : R^2 \to R^2$ be the linear transformation defined by $T(x,y) = (3x+4y, 2x-5y)$. Find $[T]_s$, when (i) $S = \{(1,0), (0,1)\}$; (ii) $S = \{(1,2), (2,3)\}$.

13. Let f_1 $(x_1, x_2, x_3, x_4) = x_1 + 2x_2 + 2x_3 + x_4$; f_2 $(x_1, x_2, x_3, x_4) = 2x_2 + x_4$; f_3 $(x_1, x_2, x_3, x_4) = -2x_1$ $4x_3 + 3x_4$ be three linear functionals on $\mathbb{R}^4$. Find the subspace which these functionals annihilate .

14. Let A be the (real) 3 × 3 matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$. Find the characteristics values and characteristic vectors of A.

Unit 3

15. Let T be a linear operator on a finite-dimensional space V. If T is diagonalizable and if $c_1,\dots,c_k$ are the distinct characteristic values of T, then show that there exist linear operators $E_1, \dots, E_k$ on V such that :

(i) $T = c_1 E_1 + \dots + c_k E_k$.

(ii) $I = E_1 + \dots + E_k$.

(iii) $E_i E_j = 0, i \neq j$.

(iv) $E_i^2 = E_i$ (E_i is a projection).

(v) the range of E_i is the characteristic space for T associated with c_i.

16. Let V be a real or complex vector space with an inner product. Prove that :

$\| \alpha + \beta \|^2 + \| \alpha - \beta \|^2 = 2\| \alpha \|^2 + 2\| \beta \|^2$ for every $\alpha, \beta \in V$.

17. Let W be a finite-dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto $W, W^{\perp}$ is the null space of E, and $V = W \oplus W^{\perp}$.

(6 × 2 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question has weightage 5.

18. If W_1 and W_1 are finite dimensional subspace of a vector space V , then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

19. (a) Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)\dots(x - c_k)$ where $c_1, \dots, c_k$ are distinct elements of F.

(b) Define T-conductor of α into W.

20. Let V and W be finite-dimensional vector spaces over the field F. Let B be an ordered basis for V with dual basis B^*, and let B' be an ordered basis for W with dual basis B'^*.

Let T be a linear transformation from V into W; let A be the matrix of T relative to B, B' and let C be the matrix of T^t relative to B'^*, B^*. Then show that $C_{ij} = A_{ji}$.

21. Let W be a subspace of an inner product space V and let $\beta \in V$. Then prove that,

   (i) The vector $\alpha \in W$ is a best approximation to $\beta \in V$ by vectors in W if and only if $\beta - \alpha$ is orthogonal to every vector in W.

   (ii) If a best approximation to $\beta \in V$ by vectors in W exists, it is unique.

   (ii) If W is finite-dimensional and $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is any orthonormal basis for W, then the vector $\alpha = \sum_{k=1}^{n} \frac{(\beta/\alpha_k)}{\|\alpha_k\|^2} \alpha_k$ is the (unique) best approximation to β by vectors in W.

(2 × 5 = 10 weightage)

D 13148 (Pages : 2) Name.................................

Reg. No...............................

FIRST SEMESTER M.A./M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2020/2021

(CBCSS)

Mathematics with Data Science

MTD 1C 01—ALGEBRA

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section/Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section/sub part/sub division may be ignored.*
4. *There will be an overall ceiling for each Section/Part that is equivalent to the maximum weightage of the Section/Part.*

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Find the number of abelian groups, up to isomorphism, of order 360.
2. Find the order of the element $5 + \langle 4 \rangle$ in the factor group $\mathbb{Z}_{12} / \langle 4 \rangle$.
3. Find all Sylow 3 -subgroups of S_4.
4. Show that $f(x) = x^2 + 8x - 2$ is irreducible over Q.
5. Find the reduced word corresponding to $a^2b^{-1}b^3a^3c^{-1}c^4b^{-2}$.
6. Find all prime and maximal ideals of $\mathbb{Z}_6$.
7. Prove that squaring the circle is impossible.
8. Show that $1 + i$ is algebraic over $\mathbb{Q}$.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions, choosing* **two** *questions from each unit.*
Each question carries a weightage 2.

Unit I

9. Prove that if m divides the order of a finite abelian group G, then G has a subgroup of order m.

10. State and prove the fundamental homomorphism theorem for groups.

11. Prove that a subgroup M is a maximal normal subgroup of a group G if and only if G/M is simple.

Unit II

12. Let H be a p-subgroup of a finite group G, then prove that :

$$(N[H]:H) \equiv (G:H) \pmod{p}.$$

13. Prove that no group of order 30 is simple.

14. Let $f(x) \in F[x]$ be of degree 2 or 3, then prove that $f(x)$ is reducible over F if and only if it has a zero in F.

Unit III

15. Prove that if F is a field, every ideal in $F[x]$ is principal.

16. Construct a field having four elements.

17. Prove that, a finite field of p^n elements exists for every prime power p^n.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.

19. Let G be a group and X be a G-set and $x \in X$, then prove that $|Gx| = (G:G_x)$, where Gx is the orbit of x and G_x is the isotropy subgroup of x.

20. State and Prove Cauchy's Theorem.

21. Prove that, if F_i is a field for $i = 1, 2, 3, \ldots r$ and F_{i+1} is a finite extension of F_i, then F_r is a finite extension of F_1, and $[F_r : F_1] = [F_r : F_{r-1}] [F_{r-1} : F_{r-2}] \ldots [F_2 : F_1]$.

(2 × 5 = 10 weightage)