D 11682-A

(Pages : 5)

Name....................................

Reg. No..................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

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INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

MTH 3E 04—PROBABILITY THEORY

Multiple Choice Questions :

1. A continuous variable can take any values within its ——————.

   (A) Range.
   (B) Co-domain.
   (C) Domain.
   (D) All of the above.

2. The weight of persons in a country is a r.v. of the type :

   (A) Continuous r.v.
   (B) Discrete r.v.
   (C) Neither discrete nor continuous.
   (D) Discrete as well as continuous.

3. If X is a random variable which can take only non-negative values, then :

   (A) $E(X^2) = [E(X)]^2$.
   (B) $E(X^2) \geq [E(X)]^2$.
   (C) $E(X^2) \leq [E(X)]^2$.
   (D) None of the above.

4. If X and Y are two random variables, the covariance between the variables $aX + b$ and $cY + d$ in terms of COV(X, Y) is :

   (A) $COV(aX + b, cY + d) = COV(X, Y)$.
   (B) $COV(aX + b, cY + d) = abcd \times COV(X, Y)$.
   (C) $COV(aX + b, cY + d) = ac\, COV(X, Y) + bd$.
   (D) $COV(aX + b, cY + d) = ac\, COV(X, Y)$.

5. The outcomes of tossing a coin three times are a variable of the type :

   (A) Continuous r.v.
   (B) Discrete r.v.
   (C) Neither discrete nor continuous.
   (D) Discrete as well as continuous.

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6. A continuous r.v. X has p.d.f. $f(x) = kx$, $0 < x < 1$, the value of k = ————————.

(A) 3.

(B) 2.

(C) 1.

(D) 4.

7. If (X,Y) is an RV of the continuous type, then marginal p.d.f. of X is ————————.

(A) $f_1(x) = \int_{-\infty}^{\infty} f(x,y)\, dy.$

(B) $f_1(x) = \int_{-\infty}^{0} f(x,y)\, dy.$

(C) $f_1(x) = \int_{0}^{\infty} f(x,y)\, dy.$

(D) None of the above.

8. Let (X,Y) be an RV of the continuous type, then conditional P.D.F. of X, given Y = y defined as :

(A) $f_{X/Y}(x \mid y) = \dfrac{f(x,y)}{f_2(y)}.$

(B) $f_{Y/X}(y \mid x) = \dfrac{f(x,y)}{f_2(y)}.$

(C) $f_{X/Y}(x \mid y) = \dfrac{f(x,y)}{f_1(x)}.$

(D) $f_{Y/X}(y \mid x) = \dfrac{f(x,y)}{f_1(x)}.$

9. Two RVs X and Y, of the continuous type are independent if and only if :

(A) $f(x,y) = f_1(x) + f_2(y).$

(B) $f(x,y) = f_1(x) - f_2(y).$

(C) $f(x,y) = f_1(x) / f_2(y).$

(D) $f(x,y) = f_1(x) f_2(y).$

10. If $f(x)$ is a probability density function of a continuous random varibale then ————————.

(A) $\int_{-\infty}^{\infty} f(x)\,dx = 0.$

(B) $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

(C) $\int_{-\infty}^{\infty} f(x)\,dx > 1.$

(D) $\int_{-\infty}^{\infty} f(x)\,dx < 0.$

11. The expected value of the constant b is :

(A) 0.

(B) 1.

(C) $1/b$.

(D) b.

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12. For constants a and b, the expected value of $aX + b$ is ————.

(A) $E(aX+b) = aX$.
(B) $E(aX+b) = b$.
(C) $E(aX+b) = aE(X)+b$.
(D) $E(aX+b) = aX + E(b)$.

13. Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then :

(A) $X_n + Y_n \xrightarrow{L} X + c$.
(B) $X_n + Y_n \xrightarrow{L} X + Y$.
(C) $X_n + Y_n \xrightarrow{L} X$.
(D) $X_n + Y_n \xrightarrow{L} XY$.

14. Which statement is/are true ?

(A) Convergence in probability implies mutual convergence in probability.
(B) Mutual convergence in probability implies convergence in probability.
(C) Sequence of random variables cannot converge in probability to two essentially different random variables.
(D) (A), (B) and (C) are true.

15. $X_n \xrightarrow{a.s.} X$:

(A) Implies $X_n \xrightarrow{P} X$ and conversly.
(B) Implies $X_n \xrightarrow{P} X$ and converse is not true.
(C) Does not implies $X_n \xrightarrow{P} X$.
(D) None of the above.

16. Suppose X is some r.v., then $X_m - X_n \xrightarrow{P} 0 \Leftrightarrow$:

(A) $X_m \xrightarrow{P} X$.
(B) $X_m \xrightarrow{P} 0$.
(C) $X_n \xrightarrow{P} 0$.
(D) $X_n \xrightarrow{P} X$.

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17. If the X_n's are identically distributed and pairwise uncorrelated with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2 < \infty$, we can choose centering constants (A_n) and norming constants (B_n) as :

(A) $A_n = \mu$ and $B_n = \sigma^2$.
(B) $A_n = \mu$ and $B_n = \sigma$.
(C) $A_n = n\mu$ and $B_n = n\sigma$.
(D) $A_n = n\mu$ and $B_n = n\sigma^2$.

18. Weak law of large numbers indicates :

(A) Convergence in probability.
(B) Convergence in distribution.
(C) Almost sure convergence.
(D) None of the above.

19. Choose the correct assumption of the central limit theorem :

(A) Samples should be dependent of each other.
(B) Samples should be independent of each other.
(C) Sampling is done with replacement.
(D) Samples are taken subjectively.

20. The Central Limit Theorem provides information about :

(A) The conditions under which sample moments converge to population moments irrespective of sample size.
(B) The conditions under which sample moments converge to population moments as sample size increases.
(C) The rate at which sample moments converge to population moments as sample size increases.
(D) None of the above.

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Discuss different types of random variables.
2. What are the properties of a distribution function ?
3. Define characteristic function of a random variable and give any two properties.
4. With a suitable example explain the concept of a bivariate random variable.
5. What do you understand by conditional expectation ? Give its properties.
6. Define convergence in probability of a random variable.
7. What do you meant by week law of large numbers ?
8. Define stable distribution.

(8 × 1 = 8 weightage)

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Part B

Answer any **six** *questions choosing two from each unit.*
Each question has weightage 2.

Unit 1

9. Expectation of a random variable always exists. Justify your answer.

10. A random variable has pgf $P(s) = \frac{1+2s+3s^2}{6}$. Obtain its pmf and mean.

11. Let h (X) be a non negative Borel measurable function of a random variable X. If E [h (X)] exits, then show that for every $\in > 0, P[h(X) \geq \in] \leq \frac{E[h(X)]}{\in}$.

Unit 2

12. Let X be a random variable with pmf $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2,\ldots,\lambda > 0$. Suppose the value X = 0 cannot be observed. Find the pmf of the truncated random variable.

13. Let X_1, X_2 be iid random variables with common pmf $P[X = x] = \frac{1}{2}, x = -1,1$. If $X_3 = X_1, X_2$, show that X_1, X_2 X_3 are pair wise independent but not independent.

14. Let X and Y be independent random variables with respective pdf f and g and distribution functions F and G. Then show that $P(X < Y) = \int_{-\infty}^{\infty} F(y)\, g(y)\, dy$.

Unit 3

15. Let $\{X_n\}$ be a sequence of random variables defined by $P[X_n = 0] = 1 - \frac{1}{n^r}$ and $P[X_n = n] = \frac{1}{n^r}$, $r > 0$ $n = 1,2,\ldots$ Examine (a) $X_n \xrightarrow{r} 0$; (b) $X_n \xrightarrow{p} 0$.

16. Let $X_1, X_2, \ldots$ be iid random variables with pdf $f(x) = \frac{\alpha+1}{x^{\alpha+2}}, x \geq 1, \alpha > 0$. Does week law of large numbers hold for $\{X_n\}$.

17. For what values of α does the strong law of large numbers hold for the sequence $\{X_k\}$ with $P[X_k = \pm k^\alpha] = \frac{1}{2}$.

(6 × 2 = 12 weightage)

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Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. For the distribution defined by the pdf $f(x) = \frac{\beta\alpha^{\beta}}{x^{\beta+1}}, x \geq \alpha,$ show that moments of order n exists if and only if $n < \beta$. If $\beta > 2$, find the mean and variance of the distribution.

19. For the distribution with pdf $f(x) = \frac{1}{2\lambda}\exp\left\{-\frac{|x-\mu|}{\lambda}\right\}, -\infty < x < \infty, \lambda > 0, -\infty < \mu < \infty,$ obtain the moment generating function. Hence obtain its mean and variance.

20. Let X and Y be two independent random variables each having pdf respectively :

$$f_1(x) = \frac{1}{\Gamma(\alpha_1)\beta^{\alpha_1}}\exp\left\{-\frac{x}{\beta}\right\}x^{\alpha_1-1}, x > 0, \alpha_1, \beta > 0 \text{ and } f_2(y) = \frac{1}{\Gamma(\alpha_2)\beta^{\alpha_2}}\exp\left\{-\frac{x}{\beta}\right\}y^{\alpha_2-1}, y > 0, \alpha_2, \beta > 0.$$

Show that X + Y and $\frac{X}{X+Y}$ are independent.

21. State and prove Slutsky's theorem.

(2 × 5 = 10 weightage)

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THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3E 03—MEASURE AND INTEGRATION

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :									FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

INSTRUCTIONS TO THE CANDIDATE

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MTH 3E 03—MEASURE AND INTEGRATION

Multiple Choice Questions :

1. Let X and Y be two topological spaces and $f : X \to Y$ be a continuous map then which of the following is necessarily true ?
   - (A) $f^{-1}(V)$ is an open set in X for every closed set V in Y.
   - (B) $f(V)$ is an open set in Y for every open set V in X.
   - (C) $f^{-1}(V)$ is an open set in X for every open set V in Y.
   - (D) None of the above options are true.
2. Let X be a non-empty set, (X, τ_1) be the discrete topological space, (X, τ_2) be a topological space, then :
   - (A) There exists no continuous map $f : X \to X$.
   - (B) There exists continuous map $f : X \to X$ depending on τ_2.
   - (C) There exists exactly one continuous map $f : X \to X$.
   - (D) Any map $f : X \to X$ is continuous.
3. Let μ be a positive measure on a σ-algebra $\mathfrak{M}$. Then :
   - (A) $\mu(\phi) \neq 0$.
   - (B) $\mu(A_1 \cup A_2 \cup \ldots \cup A_n) < \mu(A_1) + \ldots + \mu(A_n)$ if $A_1, A_2, \ldots, A_n$ are pairwise disjoint members of $\mathfrak{M}$.
   - (C) $A \subset B$ implies $\mu(A) \geq \mu(B)$ if $A, B \in \mathfrak{M}$.
   - (D) $\mu(A_n) \to \mu(A)$ as $n \to \infty$ if $A = \bigcup_{n=1}^{\infty} A_n, A_n \in \mathfrak{M}$, and $A_1 \subset A_2 \subset \ldots$.
4. Let μ be a positive measure on a σ-algebra $\mathfrak{M}$. Then choose false statement :
   - (A) $A \subset B$ implies $\mu(A) \geq \mu(B)$.
   - (B) $\mu(\phi) \neq 0$.
   - (C) μ satisfies finite additivity.
   - (D) μ satisfies monotonicity.
5. Let $\mathbb{Q}$ be the set of all rational numbers then Lebesgue measure of $\mathbb{Q} \cap [0, 1]$ is :
   - (A) 0.
   - (B) 1.
   - (C) 2.
   - (D) ∞.
6. Let $A, B \in \mathfrak{M}$ and $A \subset B$, also given that $\mu(B) = 1$. Then which of the following are necessarily true ?
   - (A) $\mu(A) = 1$.
   - (B) $\mu(A) > 1$.
   - (C) $0 \leq \mu(A) < 1$.
   - (D) $0 \leq \mu(A) \leq 1$.
7. Let $F \subset K$ in a topological space. Then which of the following are true :
   - (A) If K compact then F is compact.
   - (B) If K is compact and F is closed then F is compact.
   - (C) F compact implies K compact.
   - (D) If F is compact and closed then K is compact.
8. Which of the following are false ?
   - (A) Compact subsets of Hausdorff spaces are closed.
   - (B) Let F is closed and K is a compact set in a Hausdorff space then $F \cap K$ is compact.
   - (C) Let F is compact and K is closed in a topological space with $K \subset F$ then K is compact.
   - (D) Closed subsets of Hausdorff spaces are compact.

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9. Let f and g be real (or extended real) functions on a topological space. If $\{x : f(x) > \alpha\}$ and $\{x : g(x) < \alpha\}$ are open for every real α, then :
   (A) f is upper semicontinuous and g is upper semicontinuous.
   (B) f is upper semicontinuous and g is lower semicontinuous.
   (C) f is lower semicontinuous and g is upper semicontinuous.
   (D) f is lower semicontinuous and g is lower semicontinuous.

10. Choose the false statement :
   (A) Characteristic functions of open sets are lower semicontinuous.
   (B) Characteristic functions of closed sets are lower semicontinuous.
   (C) The supremum of any collection of lower semicontinuous functions is lower semicontinuous.
   (D) The infimum of any collection of upper semicontinuous functions is upper semicontinuous.

11. Let X be a set, $\mathfrak{M}$ be a σ-algebra on X and μ be a real measure on $\mathfrak{M}$. For every partition $\{E_i\}$ of any set $E \in \mathfrak{M}$, define $|\mu|(E) = \sup \sum_{i=1}^{\infty} |\mu(E_i)|$. Then choose the incorrect statement :
   (A) $\frac{|\mu| + \mu}{2}$ is a positive measure on $\mathfrak{M}$. (B) $\frac{|\mu| - \mu}{2}$ need not be a positive measure on $\mathfrak{M}$.
   (C) $\frac{|\mu| + \mu}{2}$ is called positive variation of μ. (D) $\frac{|\mu| - \mu}{2}$ is called negative variation of μ.

12. Suppose μ is positive measure on a σ-algebra $\mathfrak{M}$, $g \in L^1(\mu)$, and $\lambda(E) = \int_E g\, d\mu$ for $E \in \mathfrak{M}$.
   Then :
   (A) $|\lambda|(E) \neq \int_E |g|\, d\mu$, for $E \in \mathfrak{M}$. (B) $|\lambda|(E) \leq \int_E |g|\, d\mu$, for $E \in \mathfrak{M}$.
   (C) $|\lambda|(E) \geq \int_E |g|\, d\mu$, for $E \in \mathfrak{M}$. (D) $|\lambda|(E) = \int_E |g|\, d\mu$, for $E \in \mathfrak{M}$.

13. If $\mu = \lambda_1 - \lambda_2$, where λ_1 and λ_2 are positive measures, then which of the following is necessarily true ?
   (A) $\lambda_1 = \mu^+$ and $\lambda_2 = \mu^-$. (B) $\lambda_1 \neq \mu^+$ and $\lambda_2 \leq \mu^-$.
   (C) $\lambda_1 \leq \mu^+$ and $\lambda_2 \leq \mu^-$. (D) $\lambda_1 \geq \mu^+$ and $\lambda_2 \geq \mu^-$.

14. Let μ be a positive measure, suppose $1 \leq p \leq \infty$, and let q be the exponent conjugate to p. Also let $g \in L^q(\mu)$.
   I : $\int_X f\, g\, d\mu$ is a bounded linear functional on $L^p(\mu)$.
   II : $\left\| \int_X fg\, d\mu \right\| \leq \| g \|_q$.
   Then :
   (A) Both I and II are true. (B) Both I and II are false.
   (C) I is true and II is false. (D) I is false and II is true.

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15. $1 \le p < \infty$, μ is a σ-finite positive measure on X, and ϕ is a bounded linear functional on $L^p(\mu)$. Choose the incorrect statement :

(A) Then there is a unique $g \in L^q(\mu)$, q is the exponent conjugate to p.

(B) $\phi_g(f) = \int_X fg \, d\mu$ for $f \in L^p(\mu)$.

(C) $\| \phi \| = \| g \|_q$.

(D) $L^q(\mu)$ need not be isometrically isomorphic to the dual space of $L^p(\mu)$.

16. If μ is a Complex Borel measure on X, it is clear that the mapping $f \to \int_X f \, d\mu$:

(A) Is a bounded linear functional on $C_0(X)$.

(B) Has norm greater than $|\mu|(X)$.

(C) Has norm greater than or equal to $|\mu|(X)$.

(D) Need not be a bounded linear functional on $C_0(X)$.

17. Let f be an $(\mathscr{S} \times \mathscr{T})$-measurable function on $X \times Y$, then for all $x \in X$ and $y \in Y$,

(A) f_x is $\mathscr{T}$-measurable and f^y is $\mathscr{T}$-measurable.

(B) f_x is $\mathscr{T}$-measurable and f^y is $\mathscr{S}$-measurable.

(C) f_x is $\mathscr{S}$-measurable and f^y is $\mathscr{T}$-measurable.

(D) f_x is $\mathscr{S}$-measurable and f^y is $\mathscr{S}$-measurable.

18. Let $(X, \mathscr{S}, \mu)$ and $(Y, \mathscr{T}, \lambda)$ and σ-finite measure spaces. Suppose that $Q \in \mathscr{S} \times \mathscr{T}$ and define $(\mu \times \lambda)(Q) = \int_X \lambda(Q_x) \, d\mu(x) = \int_X \mu(Q^y) \, d\lambda(y)$.

I : $\mu \times \lambda$ is σ-finite.

II : $\mu \times \lambda$ is countable subadditive not countable additive on $\mathscr{S} \times \mathscr{T}$.

(A) Both I and II are true.
(B) Both I and II are false.
(C) I is true and II is false.
(D) I is false and II is true.

19. Let $(X, \mathscr{S}, \mu)$ and $(Y, \mathscr{T}, \lambda)$ are complete measure spaces :

I : $(X \times Y, \mathscr{S} \times \mathscr{T}, \mu \times \lambda)$ is complete.

II : $\mu \times \lambda$ is a measure.

(A) Both I and II are true.
(B) Both I and II are false.
(C) I is true and II is false.
(D) I is false and II is true.

20. Let E be the cantor's set in $\mathbb{R}^1$ and m be the Lebesgue measure on $\mathbb{R}^1$. Then choose the false statement from below :

(A) E and $\mathbb{R}^1$ have same cardinality.
(B) $m(E) = 0$.
(C) $m(\mathbb{R}^1) \neq 0$.
(D) $m(\mathbb{R}^1) = m(E)$.

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3E 03—MEASURE AND INTEGRATION

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question has weightage 1.

1. If μ is a positive measure on a σ-algebra M. Then show that $\lim \mu(A_n) = \mu(\lim A_n)$, where $\{A_n\} \in M$ with $A_1 \subset A_2 \subset A_3 \ldots\ldots$

2. Let f and g are measurable functions such that $f = g$ a.e. Then prove that $\int_E f d\mu = \int_E g d\mu, \forall E \in M$

3. Prove $\phi(E) = \int_E s d\mu$ is a measure on M, where s is measurable simple function on X and $E \in M$.

4. Define Borel measure, regular measure, and σ-finite measure.

5. If $\lambda_1 << \mu$ and $\lambda_2 \perp \mu$ then prove that $\lambda_1 \perp \lambda_2$. Also prove that if $\lambda << \mu$ then $|\lambda| << \mu$.

Turn over

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6. Define total variation measure.

7. Define measurable rectangle, Elementary sets and Monotone class.

8. Give an example to show that the condition $f \in L^1(\mu \times \nu)$ cannot be dropped from Fubini's theorem to obtain the conclusion of the theorem.

(8 × 1 = 8 weightage)

Part B (Short Essays)

Answer any **two** *questions from each unit.*
Each question has weightage 2.

UNIT 1

9. State and prove Lebesgue's Dominated convergence Theorem.

10. Let $f \in L^1(\mu)$. and

(a) If $\int_E f d\mu = 0, \forall E \in M$, then show that $f = 0$ a.e. on X.

(b) If $\left| \int_E f d\mu \right| = \int_E |f| d\mu$ then show that $\exists$ a constant α such that $\alpha f = |f|$ a.e. on X.

11. State and prove Urysohn's Lemma.

UNIT 2

12. Prove that there exist a non-measurable subset of R^1.

13. State and prove the Vitali-Carathedory theorem.

14. Let μ be a complex measure on a σ-algebra M in X. Then show that there is a measurable function h such that $|h(x)| = 1 \; \forall x \in X$ and such that $d\mu = hd|\mu|$.

UNIT 3

15. Define the x-section and y-section of a measurable function $f(x, y)$ and show that they are measurable with respect to respective σ-algebras.

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16. Let $[X, \zeta, \mu]$ and $[Y, \tau, \nu]$ be σ-finite measure spaces, and f be an $\zeta \times \tau$–measurable function on $X \times Y$ write $\phi(x) = \int_Y f_x \, d\nu$ and $\psi(y) = \int_X f^y \, d\mu$ for $x \in X$ and $y \in Y$. Then prove ϕ is ζ-measurable and ψ is τ-measurable and $\int_X \phi d\mu = \int_Y \psi d\nu = \int_{X \times Y} f d(\mu \times \nu)$.

17. Show that there exist a set which is not measurable but its x-section and y-section are measurable.

(6 × 2 = 12 weightage)

Part C (Essays)

Answer any **two** *questions.*
Each question has weightage 5.

18. State and prove the completion theorem for measure space (X, M, μ).

19. State and prove the theorem of Lebesgue-Radon-Nikodym.

20. Let $[X, S]$ and $[Y, \Gamma]$ be two measurable spaces. Show that $S \times \Gamma$ is the smallest monotone class containing all elementary sets.

21. Let $[X, \zeta, \mu]$ and $[Y, \tau, \nu]$ be σ-finite measure space. For $Q \in \zeta \times \tau$ write $\phi(x) = \nu(Q_x)$ and $\psi(y) = \mu(Q^y)$ for $x \in X$ and $y \in Y$. Then ϕ is ζ-measurable and ψ is τ–measurable and $\int_X \phi d\mu = \int_Y \psi d\nu$. Also show that the σ-finite condition cannot be dropped.

(2 × 5 = 10 weightage)

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D 11680–A (Pages : 4) Name.....................................

Reg. No..................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3E 02—CRYPTOGRAPHY

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

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INSTRUCTIONS TO THE CANDIDATE

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5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

MTH 3E 02—CRYPTOGRAPHY

Multiple Choice Questions :

1. The ———— is the original message before transformation.

   (A) Ciphertext. (B) Plaintext.

   (C) Secret-text. (D) None of the above.

2. d_K ———— algorithm transforms ciphertext to plaintext.

   (A) Encryption. (B) Decryption.

   (C) Either (A) *or* (B). (D) Neither (A) *or* (B).

3. The shift cipher sometimes referred to as the :

   (A) Shift modulus. (B) Cipher modulus.

   (C) Caeser cipher. (D) None of these.

4. Let $P = C = K = Z_{26}$. Then how many possible shift cipher is possible ?

   (A) 26!. (B) 2^{26}.

   (C) 52. (D) 26.

5. Let $P = C = K = Z_{26}$. $e_{18}(18) = 10$ then what is $d_{18}(10) = ?$

   (A) 18. (B) 10.

   (C) 0. (D) 2.

6. Let $P = C = K = Z_{26}$. K consists of all possible permutations of the 26 symbols 0, 1, ..., 25. Let $\pi \in K$ and $e_\pi(2) = 3$ then what is $d_\pi(3)$.

   (A) 0. (B) 1.

   (C) 2. (D) 3.

7. $ax \equiv b \pmod{m}$ has a unique solution in $\mathbb{Z}_m$ if ———— ?

   (A) $a \mid d$. (B) $g.c.d.(a, m) = 1$.

   (C) $g.c.d.(a, m) = b$. (D) $g.c.d.(a, b) = 1$.

8. If P is a prime then ϕ (P) is ?

   (A) 1. (B) P – 1.

   (C) P. (D) None of the above.

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9. Let $P = C = Z_{26}$ and let $K = (a, b)$ then the decrypted affine cipher $d_k(x)$ is ———— ?

(A) $d_K(x) = a^{-1}(x+b) \pmod{26}$.
(B) $d_K(x) = a^{-1}(x-b) \pmod{26}$.
(C) $d_K(x) = -ax + b \pmod{26}$.
(D) $d_1(x) = -ax - b \pmod{26}$.

10. Let $P = C = \mathbb{Z}_{26}$ and let $K = (5,1)$ then the encrypted affine cipher $d_K(6)$ is :

(A) 1.
(B) 2.
(C) 3.
(D) 5.

11. $P = C = K = (\mathbb{Z}_{26})^m$ and $K = (k_1, k_2, ..., k_m)$ then the decrypted vigenere cipher

$d_K(x_1, x_2, ..., x_m) =$ ———— ?

(A) $(x_1 - k_1, x_2 - k_2, ..., x_m - k_m)$.

(B) $(x_1 + k_1, x_2 + k_2, ..., x_m + k_m)$.

(C) $\left(x_1 - k_1, x_2 + k_2, ..., x_m + (-1)^m k_m\right)$.

(D) $\left(x_1 + k_1, x_2 - k_2, ..., x_m + (-1)^{m+1} k_m\right)$.

12. $P = C = K = (\mathbb{Z}_{26})^3$ and $K = (5, 10, 15)$ then the decrypted vigenere cipher

$e_K(10, 18, 21) =$ ———— ?

(A) (15, 2, 11).
(B) (15, 28, 36).
(C) (15, 2, 10).
(D) (5, 8, 6).

13. Let $P = C = (\mathbb{Z}_{26})^m$ and let K be a $m \times m$ invertible matrix over $\mathbb{Z}_{26}$ then the decrypted hill cipher

$d_K(x)$ is ———— ?

(A) $d_K(x) = xK$.
(B) $d_K(x) = Kx$.
(C) $d_K(x) = xK^{-1}$.
(D) $d_K(x) = K^{-1}x$.

14. I. Matrix multiplication is commutative.

II. Matrix multiplication is associative.

(A) I is true and II is false.
(B) I is false and II is true.
(C) I and II are false.
(D) I and II are true.

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15. Let $P = C = (\mathbb{Z}_{26})^m$ and let K be a permutation of the set $\{1, 2, ..., m\}$ then the encrypted permutation cipher $e_K(x_1, x_2, ..., x_m)$ is ———— ?

(A) $\left(x_{\pi(1)}, x_{\pi(2)}, ..., x_{\pi(m)}\right)$.

(B) $\left(x_{\pi^{-1}(1)}, x_{\pi^{-1}(2)}, ..., x_{\pi^{-1}(m)}\right)$.

(C) $\left(x_{\pi(m)}, x_{\pi(m-1)}, ..., x_{\pi(1)}\right)$.

(D) $\left(x_{\pi^{-1}(m)}, x_{\pi^{-1}}\ (m-1), ..., x_{\pi^{-1}(1)}\right)$.

16. Let $P = C = (\mathbb{Z}_{26})^3$ and let K be a permutation of the set $\{1, 2, ..., m\}$ then the decrypted permutation cipher $d_K(x_1, x_2, ..., x_m)$ is ———— ?

(A) $\left(x_{\pi(1)}, x_{\pi(2)}, ..., x_{\pi(m)}\right)$.

(B) $\left(x_{\pi^{-1}(1)}, x_{\pi^{-1}(2)}, ..., x_{\pi^{-1}(m)}\right)$.

(C) $\left(x_{\pi(m)}, x_{\pi(m-1)}, ..., x_{\pi(1)}\right)$.

(D) $\left(x_{\pi^{-1}(m)}, x_{\pi^{-1}}\ (m-1), ..., x_{\pi^{-1}(1)}\right)$.

17. Let $P = C = \left(\mathbb{Z}_{26}\right)^3$ and let $K = (1, 2, 3)$ be a permutation of the set $\{1, 2, 3\}$ then the dcrypted permutation cipher $d_K(4, 5, 6)$ is ———— ?

(A) (4, 5, 6).

(B) (1, 2, 3).

(C) (5, 6, 4).

(D) (6, 4, 5).

18. Consider a random throw of a pair of dice. What is the probability that the sum is 4 ?

(A) $\frac{1}{6}$.

(B) $\frac{4}{6}$.

(C) $\frac{1}{36}$.

(D) $\frac{3}{36}$.

19. If X and Y are independent random variable then which of the following is true ?

(A) $Pr[x, y] \leq Pr[x] Pr[y]$.

(B) $Pr[x, y] \geq Pr[x] Pr[y]$.

(C) $Pr[x, y] \neq Pr[x] Pr[y]$.

(D) $Pr[x, y] = Pr[x] Pr[y]$.

20. A family has 2 children. Given that one of the children is a boy, what is the probability that the other child is also a boy ?

(A) $\frac{1}{2}$.

(B) $\frac{1}{3}$.

(C) $\frac{3}{4}$.

(D) $\frac{1}{4}$.

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D 11680 **(Pages : 3)** Name.................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3E 02—CRYPTOGRAPHY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
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4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries weightage 1.

1. Define Euler phi-function and Affine Cipher. Find the number of keys in the Affine Cipher over Z_{30}.
2. Prove that $-a \bmod m = m - (a \bmod m)$ where $a, m > 0$ and $a \neq 0 \pmod{m}$.
3. Decrypt the message XHPEN BDYQ which was produced with Auto key Cipher having key K = 16.
4. Define concave function and show that the function $f(x) = x^2$ is concave in the interval $(-\infty, \infty)$.
5. Explain entropy and redundancy of a natural language.
6. Describe block ciphers with example.
7. State the Piling up Lemma.

Turn over

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8. Define Strongly Universal Hash Families. Prove that for such a (N, M) hash family

$$(X, Y, \kappa, H), \left|\{K \in \kappa : h_\kappa(x) = y\}\right| = \frac{|\kappa|}{M}, \forall x \in X, y \in Y.$$

(8 × 1 = 8 weightage)

Part B (Short Essays)

Answer any **two** *questions from each unit.*
Each question carries weightage 2.

Unit 1

9. List all invertible elements in Z_m for $m = 28, 35$.

10. Define involutory key. Find the number of involutory keys in the Hill Cipher over Z_{26} in the case $m = 2$.

11. Explain Cryptanalysis of LFSR Stream Cipher.

Unit 2

12. Let S is a random variable representing the sum of a pair of dice. Compute H (S).

13. Prove that the Shift Cipher achieves perfect secrecy if every key is used with equal probability 1/26.

14. Define unicity distance of a cryptosystem. Calculate it for Hill Cipher with $m \times m$ encryption matrix.

Unit 3

15. Define balanced S-box. Prove that for a balanced S-box

$$N_L(0, b) = 2^{m-1}, \forall \text{ integers } b \text{ such that } 0 < b \leq 2^n - 1.$$

16. Describe Data Encryption Standard (DES).

17. Compare keyed and unkeyed hash functions. Define (N, M) hash family. What are the three problems that should be addressed by a secure hash function.

(6 × 2 = 12 weightage)

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Part C (Essays)

Answer any **two** *questions.*
Each question carries weightage 5.

18. (a) Define Vigenere Cipher. Encrypt the message ATTACK AT ONCE using keyword READY.

    (b) Define Hill Cipher. Prove that t the number of 2×2 invertible matrices over Z_p is :

    $(p^2 - 1)(p^2 - p)$, where p prime.

19. (a) Suppose (P, C, K, E, D) is a cryptosystem where $|C| = |P|$ and keys are chosen equiprobably. Let R_L be the redundancy of the language. Then prove that given a string of cipher text of length n, the expected number of spurious keys $\bar{s}_n \geq \frac{|K|}{|P|^{nR_L}} - 1$.

    (b) Consider a cryptosystem in which P = $\{a, b, c\}$, K = $\{K_1, K_2, K_3\}$ and C = {1, 2, 3, 4}. The encryption matrix is as follows :

	a	b	c
K_1	1	2	3
K_2	2	3	4
K_3	3	4	1

Given that the keys are chosen equiprobably and the plaintext probability distribution is Pr $[a]$ = 1/2 , Pr $[b]$ = 1/3, Pr $[c]$ = 1/6. Then compute H (P), H (C), H (K), H (K/C), H (P/C).

20. Explain Linear cryptanalysis and Differential cryptanalysis.

21. Describe Message Authentication Codes (MAC) and explain the construction of Nested MAC, HMAC.

(2 × 5 = 10 weightage)

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D 11679–A **(Pages : 4)** **Name..................................**

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THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3E 01—CODING THEORY

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

INSTRUCTIONS TO THE CANDIDATE

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MTH 3E 01—CODING THEORY

Multiple Choice Questions :

1. In a BSC, bit reliability is found to be 0.8. If the transmitted word is v = 101101, what is the probability that the received word is w = 110011 ?

   (A) 0.0019. (B) 0.2021.

   (C) 0.0102. (D) None of these.

2. The maximum value of distance between any *two* codewords u and v, ie $d(u, v)$ will be :

   (A) $wt(u + v)$. (B) $0.5 \times wt(u + v)$.

   (C) $d(u, w) + d(w, v)$. (D) $0.5 \times [d(u, w) + d(v, w)]$.

3. A communication channel has bit reliability 0.8. Output code word is decoded as 000 or 111 depending on majority of 1's in the received 3 bits of the output. If the transmitted codeword is 111, what is the probability that it is decoded as 111 ?

   (A) 1. (B) 0.972.

   (C) 0.896. (D) None of these.

4. Given following codewords :

   C1 = {00000, 11111} ; C2 = {00000, 11100, 00111, 11011} ; C3 = {00000, 11110, 01111,10001}

   C4 = {000000, 101010, 010101, 111111}. Choose the best option.

   (A) C1, C2 are linear codes. (B) C1, C2, C4 are linear codes.

   (C) C3,C4, C4 are linear codes. (D) All linear codes.

5. The dual code $C^{\perp}$ for the code C = <S> where S = {1010, 0101, 1111} will be :

   (A) {0011, 0101, 1010, 1111}. (B) {0000, 0101, 1010, 1111} .

   (C) {0000, 0110, 1010, 1111}. (D) None of these.

6. Given S = {0110, 1010, 1100, 0011, 1111}. Is S linearly dependent or independent set ?

   (A) Dependent.

   (B) Independent.

   (C) Information inadequate to determine.

   (D) None of these.

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7. The basis of subspace {0} is :

(A) {1}.
(B) ϕ.
(C) Having dimension 1.
(D) None of these.

8. The linear code K^5 has ———— possible diffferent bases.

(A) 5.
(B) 32.
(C) 840.
(D) 83328.

9. A linear code has dimension k and length n. The order of generator matrix is :

(A) $k \times k$.
(B) $n \times n$.
(C) $n \times k$.
(D) $k \times n$.

10. A linear code of dimension ———— has 28 different bases.

(A) 7.
(B) 3.
(C) 4.
(D) 2^{27}.

11. Assume that rows of G form a basis for C = <S> and columns of H form a basis for $C^{\perp}$ Then G.H. = ?

(A) Identity matrix.
(B) Ones matrix.
(C) Zeros matrix.
(D) None of these.

12. Consider the linear code generated by S = {10101, 01010, 00011}. Out of the following, which is not a valid codeword ?

(A) 11101.
(B) 10110.
(C) 01001.
(D) 11111.

13. A generator matrix for the code C = {0000, 1110, 0111,1001} is $G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$; corresponding parity check matrix H can be :

(A) [00, 01, 10, 11]'.
(B) [01, 11, 10, 01]'.
(C) [01, 10, 00, 11]'.
(D) None of these.

14. Consider a generator matrix G in standard form for a (7, 4, d) code. Its one row is 0 0 0 1 0 1 1. The parity-check bit sequence corresponding to information bit 0001 will be :

(A) 011.
(B) 001.
(C) 010
(D) 111.

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15. The distance of the linear code with generator matrix $G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ is :

(A) 1. (B) 2.

(C) 3. (D) 6.

16. A code C has dimension k and encoded word has 7 bits. There are ——— different cosets of C.

(A) 4. (B) 8.

(C) 16. (D) 128.

17. An upper bound on dimension k of a linear code with $n = 10$ and $d = 5$ is :

(A) 3. (B) 4.

(C) 5. (D) None of these.

18. For a linear code with $n = 23$ and $d = 7$:

(A) Redundant bits are 11, code is perfect.

(B) Perfect code not possible.

(C) Maximum weight of error vector is 4.

(D) None of these.

19. For Golay code :

(A) It is a perfect code. (B) Maximum 3 errors can be corrected.

(C) Generator matrix is 12 × 23. (D) All of these.

20. $\left(x^6 + x^7 + x^9\right) \left(\text{mod}\left(1\,x^7\right)\right)$ is :

(A) $x + x^3 + x^5$. (B) $1 + x^6 + x^7 + x^9$.

(C) $1 + x^2 + x^6$. (D) None of these.

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D 11679 (Pages : 3) Name.......................................

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3E 01—CODING THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

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Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Calculate $\phi_{.97}(v,w)$ for $v = 01101101, w = 10001110$.

2. Find all error patterns detected by the code $C = \{1000, 0100, 1111\}$.

3. Encode the message u = 0111 for the code C with generator matrix $G = \begin{bmatrix} 1000101 \\ 0100100 \\ 0010110 \\ 0001011 \end{bmatrix}$.

4. Compute an upper bound for the size of a linear code C of length $n = 6$ and $d = 3$.
5. State Gilbert-Varshamov bound.
6. Define a perfect code and give an example.

Turn over

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7. Find the quotient and reminder when $h(x) = 1 + x^5$ is divided into $f(x) = x^2 + x^3 + x^4 + x^8$.

8. Find the generator polynomial of the dual cyclic code C of length $n = 7$ and having generator polynomial $g(x) = 1 + x + x^3$.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing two from each unit.*
Each question has weightage 2.

Unit 1

9. Find a generator matrix for the code $C = \{0000, 1111, 0110, 1001\}$ in the RREF form.

10. List the cosets of the linear code with generator matrix $G = \begin{bmatrix} 101010 \\ 010101 \end{bmatrix}$.

11. Construct an SDA assuming CMLD for the code $C = \{0000, 1010, 1101, 0111\}$.

Unit 2

12. For the Hamming code of $n = 7$ decode the message $w =$ 1101001.

13. Give a generating and parity check matrices for an extended Hamming code of length $n = 8$.

14. Write generator matrices for RM (1,2) and RM (1,3).

Unit 3

15. Find four idempotent generators for the cyclic code of $n = 7$.

16. Let $g(x) = 1 + x^2 + x^3$ be a generating polynomial for cyclic code C of length $n = 7$ Find a parity check matrix.

17. Construct GF (2^4) using $h(x) = 1 + x + x^4$.

(6 × 2 = 12 weightage)

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Part C

Answer **two** *questions.*
Each question has weightage 5.

18. List all the cosets of the linear code C with generator matrix $G = \begin{bmatrix} 100110 \\ 010011 \\ 001111 \end{bmatrix}$ using this decode the message w =110110.

19. Decode the word w = 000111000111,011011010000 in the extended Golay codeC_{24}.

20. Prove that the distance of C_{24} is 8.

21. Decode in BCH code C_{15}. the received vector w = 110111101011000.

(2 × 5 = 10 weightage)

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D 11678-A (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

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MTH 3C 14—PDE AND INTEGRAL EQUATIONS

Multiple Choice Questions :

1. Find the solution of Cauchy problem $u_x = c_0 u + c_1$ with initial condition $u(0, y) = y$.

   (A) $u(x, y) = e^{c_0 x}\left[\int_0^x e^{-c_0 \xi} c_1(\xi, y)\, d\xi + y\right]$.

   (B) Solution doesnot exists.

   (C) $u(x, y) = 0$.

   (D) None of these.

2. Which of the following is not true about the solution of a Cauchy problem ?

   (A) Solution doesnot exists.
   (B) Solution unique.
   (C) Infinitely many solutions.
   (D) None of these.

3. Solution of $u_x + u_y = 2$ subject to the initial condition $u(x, 0) = x^2$ is :

   (A) $u(x, y) = 4y + (x - y)^2$.
   (B) $u(x, y) = y + (x - y)^3$.
   (C) $u(x, y) = 2y + (x - y)^2$.
   (D) $u(x, y) = 2y^2 + (x - y)^2$.

4. For a first-order quasilinear PDE to have a unique solution near the initial curve, we must have :

   (A) $J = 0$.
   (B) Transversality condition.
   (C) $\frac{\partial x}{\partial t}\frac{\partial y}{\partial s} - \frac{\partial x}{\partial s}\frac{\partial y}{\partial t} = 0$.
   (D) $\begin{vmatrix} a & b \\ (x_0)_s & (y_0)_s \end{vmatrix} = 0$.

5. Let $u(x, y)$ be the solution of the Cauchy problem $xu_x + u_y = 1, u(x, 0) = 2\log x ; x > 1$. Then find $u(e, 1)$.

   (A) -1.
   (B) 0.
   (C) 1.
   (D) e.

6. Let $a, b, c, d \in \mathbb{R}$, such that $c^2 + d^2 \neq 0$ then the cauchy problem $au_x + bu_y = e^{x+y}, x, y \in \mathbb{R}$ and $u(x, y) = 0$ on $cx + dy = 0$ has a unique solution if :

   (A) $ac + bd \neq 0$.
   (B) $ad - bc \neq 0$.
   (C) $ac - bd \neq 0$.
   (D) $ad + bc \neq 0$.

7. The complete integral of the equation $u_x u_y = 2xy$ is the following with a and b be the arbitrary constants :

   (A) $u(x, y) = \frac{ax^2}{2} + \frac{y^2}{a} + b$.
   (B) $u(x, y) = x^2 + \frac{y^2}{2} + b$.
   (C) $u(x, y) = \frac{bx^2}{2} + \frac{y^2}{b}$.
   (D) $u(x, y) = \frac{x^2}{2} + y^2 + a$.

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8. The PDE $u_{xx} + u_{yy} = 0$ is :

(A) Hyperbolic. (B) Parabolic.

(C) Elliptic. (D) None of these.

9. The PDE $y^3 u_{xx} - (x^2 - 1) u_{yy} = 0$ is :

(A) Parabolic in $\{(x, y) : x < 0\}$. (B) Hyperbolic in $\{(x, y) : y > 0, x > 1\}$.

(C) Elliptic in $\mathbb{R}^2$. (D) Parabolic in $\{(x, y) : x > 0, y > 0\}$.

10. Pic the region in which the following PDE is hyperbolic :

(A) $xy \neq 1$. (B) $xy \neq 0$.

(C) $xy > 1$. (D) $xy > 0$.

11. Consider the Dirichlet problem in a bounded domain :

$\Delta u = f(x, y) \quad (x, y) \in D,$

$u(x, y) = g(x, y) \quad (x, y) \in \partial D.$

The problem has ——————— in $C^2(D) \cap C(\bar{D})$.

(A) Unique solution. (B) No solution.

(C) At most one solution. (D) Infinite solution.

12. Let u be a function in $C^2(D)$ satisfying the mean value property at every point in D. Then :

(A) u is harmonic in D. (B) u is harmonic in $\bar{D}$.

(C) u is harmonic in ∂D. (D) None of the above.

13. Poisson's equation become Dirichlet's problem when :

(A) It has unique solution. (B) $u(x, y) = f(x, y)$.

(C) $\partial u(x, y) = 0$. (D) $\partial u(x, y) = g(x, y)$.

14. Let u and v are harmonic function defined on open set in $\mathbb{R}^2$. Then which of the following is true ?

(A) $u + v$ is harmonic. (B) uv is harmonic.

(C) Both (A) and (B). (D) Neither (A) nor (B).

15. Which of the following equation represents a Laplace equation ?

(A) $u_{xx} + u_{tt} = 0$. (B) $u_{xx} - u_{tt} = 0$.

(C) $u_{xx} + u_t = 0$. (D) $u_{xx} - u_t = 0$.

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16. By the method of successive approximation, the solution of the integral equation

$u(x) = (1+x) - \int_0^x u(\xi)\, d\xi, u_0(x) = 1$ is :

(A) 0.
(B) 1.
(C) 2.
(D) None of these.

17. Resolvent Kernel of Fredholm integral equation

$\phi(x) = f(x) + \lambda \int_a^b K(x, \xi)\, \phi(\xi)\, d\xi$ is :

(A) $R(x, \xi; \lambda) = \sum_{n=1}^{\infty} K_n(x, \xi)$.
(B) $R(x, \xi; \lambda) = \sum_{n=1}^{\infty} \lambda^{n-1} K_n(x, \xi)$.
(C) $R(x, \xi; \lambda) = \cos(\xi - x)$.
(D) $R(x, \xi; \lambda) = K_n(x, \xi)$.

18. Solution of the integral equation :

$\sin x = \lambda \int_0^x e^{x-\xi} u(\xi)\, d\xi$

(A) $u(x) = \cosh x - \sinh x$.
(B) $u(x) = \lambda(\cos x - \sin x)$.
(C) $u(x) = \frac{1}{\lambda}(\cos x - \sin x)$.
(D) $u(x) = \cos x - \sin x$.

19. By the method of successive approximation the solution of the integral equation

$u(x) = 1 + \int_0^x (\xi - x)\, u(\xi)\, d\xi$

(A) $u(x) = \cosh x$.
(B) $u(x) = \cos x$.
(C) $u(x) = \sinh x$.
(D) $u(x) = \sin x$.

20. By the method of successive approximation the solution of the integral equation

$u(x) = 1 + \int_0^x (x - \xi)\, u(\xi)\, d\xi$

(A) $u(x) = \cosh x$.
(B) $u(x) = \cos x$.
(C) $u(x) = \sinh x$.
(D) $u(x) = \sin x$.

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D 11678 (Pages : 4) Name.......................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Solve $u_x = 1$ subject to the condition $u(0, y) = g(y)$.
2. For the equation $u_{xx} + xu_{yy} = 0, x > 0$ find a canonical transformation $q = q(x, y), r = r(x, y)$ and the corresponding canonical form.
3. Derive the general solution of one dimensional wave equation.
4. Show that the only possible value for the eigen value problem $\frac{d^2X}{dx^2} + \lambda X = 0, 0 < x < L, X(0) = X(L) = 0$ are positive real numbers.
5. State mean value principle property of harmonic functions.

Turn over

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6. Show that the Dirichlet problem has atmost one solution in a smooth domain.

7. Explain the volterra equation of third kind ? Can the same be transformed to that of second kind ? Justify.

8. If $y'' = F(x)$ and y satisfies the initial condition $y(0) = y_0, y'(0) = y'_0$. Show that

$$y(x) = \int_0^x (x - \xi) F(\xi) d\xi + y'_0 x + y_0.$$

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

Unit 1

9. Solve the equation $-yu_x + xu_y = u$ subject to the initial condition $u(x, 0) = \psi(x)$.

10. Show that the Cauchy problem $u_x + u_y = 1, u(x. x) = x$ has uniquely many solutions.

11. Explain and justify the well posedness of Cauchy problem.

Unit 2

12. Solve $u_t - 17 u_{xx} = 0, 0 < x < \pi, t > 0$

$$u(0, t) = u(\pi, t) = 0, t \geq 0$$

$$u(x, 0) = f(x) = \begin{cases} 0, & 0 \leq x \leq \dfrac{\pi}{2} \\ 2, & \dfrac{\pi}{2}, \leq x \leq \pi \end{cases}.$$

13. Show that the Neumann problem for the vibrating string $u_{tt} - c^2 u_{xx} = F(x, t), 0 < x < L, t > 0$ subject to the conditions

$$u_x(0, t) = a(t), u_x(L, t) = b(t), t \geq 0$$

$$u(x, 0) = f(x), 0 \leq x \leq L$$

$u_t(x, 0) = g(x), 0 \leq x \leq L,$ has unique solution.

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14. State and prove the weak maximum principle.

Unit 3

15. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real.

16. Show that $y'' + Ay' + By = 0,\ y(0) = y(1) = 0,$ where A and B are constants, leads to the equation

$$y(x) = \int_0^1 k(x, \xi)\, y(\xi)\, d\xi, \text{ where } k(x, \xi) = \begin{cases} B\xi(1 - x) + Ax - A, & \xi < x \\ Bx(1 - \xi) + Ax, & \xi > x \end{cases}.$$

17. Formulate the integral equation corresponding to the differential equation $x^2 y'' + xy' + (\lambda x^2 - 1)\, y = 0.$

(6 × 2 = 12 weightage)

Part C

Answer **two** *questions.*
Each question has weightage 5.

18. Use the method of characteristic strips to solve the non-linear eikonal equation $p^2 + q^2 = n_0^{\,2}$ subject to the condition $u(x, 2x) = 1.$

19. For the problem $u_{tt} - 4u_{xx} = 0,\ -\infty < x < \infty,\ t > 0$ with initial conditions

$$u(x, 0) = f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$u_t(x, 0) = \begin{cases} 4, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

a) Find $u(x, 1)$.

b) Find $\lim_{t \to \infty} u(5, t)$.

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c) Find the set of all points where the solution is singular.

d) Find the set of all points where the solution is continuous.

20. a) Using the separation of variables method find a (formal) solution of a vibrating string with fixed ends :

$$u_{tt} - c^2 u_{xx,} = 0, 0 < x < L, 0 < t$$
$$u(0, t) = u(L, t) = 0, t \geq 0$$
$$u(x, 0) = f(x)\ 0 \leq x \leq L,$$
$$u_t(x, 0) = g(x)\ 0 \leq x \leq L.$$

(b) Prove that the above solution can be represented as a superposition of a forward and a backward wave.

21. Determine the resolvent kernel of $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$ where $k(x, \xi) = 1 - 3x\xi$.

(2 × 5 = 10 weightage)

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D 11677–A **(Pages : 4)** **Name.....................................**

Reg. No..................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3C 13—FUNCTIONAL ANALYSIS

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

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MTH 3C 13—FUNCTIONAL ANALYSIS

Multiple Choice Questions :

1. If E is finite dimensional linear space of dimension n, and F is a subset of E with m elements, where $m < n$, then which of the following is true ?

   (A) F can span E.
   (B) F is linearly independent in E.
   (C) F is linearly dependent in E.
   (D) F can not be a basis of E.

2. Which of the following is not a linear space over $\mathbb{R}$?

   (A) $\mathbb{C}$.
   (B) $\mathbb{R}$.
   (C) $\mathbb{Q}$.
   (D) None of these.

3. Dimension of $\mathbb{C}^n$ as a linear space over $\mathbb{C}$ is :

   (A) n.
   (B) $n+1$.
   (C) n^2.
   (D) $2n$.

4. Which of the following linear space is infinite dimensional ?

   (A) $\mathbb{R}$ over $\mathbb{Q}$.
   (B) $\mathbb{Q}$ over $\mathbb{Q}$.
   (C) $\mathbb{C}$ over $\mathbb{C}$.
   (D) $\mathbb{C}$ over $\mathbb{R}$.

5. A linear map $A : E_1 \to E_2$ between two linear spaces is an isomorphism if :

   (A) $\ker A = \{0\}$ and $\operatorname{Im}A = E_2$.
   (B) $\ker A \neq \{0\}$ and $\operatorname{Im}A = E_2$.
   (C) $\ker A = \operatorname{Im}A$.
   (D) $\ker A = E_1$ and $\operatorname{Im}A = E_2$.

6. Which of the following is not a property of norm in general ?

   (A) $\|x\| \geq 0$.
   (B) $\|x+y\| \leq \|x\| + \|y\|$.
   (C) $\|kx\| = k\|x\|$.
   (D) $\|x\| = 0$ iff $x = 0$.

7. A complete normed space is known as a

   (A) Hilbert space.
   (B) Compact space.
   (C) Banach space.
   (D) Euclidean space.

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8. Which of the following is a Banach space ?

(A) $P[a, b]$ with supremum norm. (B) $C[a, b]$ with supremum norm.

(C) s^* with supremum norm. (D) $C[a, b]$ with p-norm.

9. The term Hilbert space stands for a :

(A) Complete inner product space. (B) Compact linear space.

(C) Complete normed space. (D) Complete metric space.

10. For any bounded linear operator $A : X \to Y$, ker A is :

(A) A closed subspace of Y. (B) An open subspace of Y.

(C) A closed subspace of X. (D) An open subspace of X.

11. A bijective map $A : X \to Y$ is open if and only if :

(A) $A : X \to Y$ is invertible. (B) $A : X \to Y$ is bounded.

(C) $A^{-1} : Y \to X$ is bounded. (D) $A^{-1} : Y \to X$ is open.

12. If $\{A_n\}$ is a sequence of operators on a normed space X, then $A_n \to A$ strongly if and only if :

(A) $A_n x \to A_x$ for all $x \in X$.

(B) $\|A_n - A\| \to 0$ as $n \in \infty$.

(C) $f(A_n x) \to f(Ax)$ for all $x \in X$ and for all $f \in X^*$.

(D) None of these.

13. Let M be a closed subspace of a normed space N. Then the quotient space N/M is a Banach space if and only if :

(A) M is a Banach space. (B) N is a Banach space.

(C) N = M. (D) None of these.

14. Which of the following normed space is not separable ?

(A) $\left(l_\infty, \|\cdot\|_\infty\right)$. (B) $\left(l_p, \|\cdot\|_p\right), 1 \le p < \infty$.

(C) $\left(\mathbb{C}^n, \|\cdot\|_p\right), 1 \le p < \infty$. (D) $\left(\mathbb{C}^n, \|\cdot\|_\infty\right)$.

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15. Which of the following is not true ?

(A) The space c_0 is a closed subspace of l_∞.

(B) The space s^* is a closed subspace of l_∞.

(C) The space c is a closed subspace of l_∞.

(D) The space P [0, 1] is not closed in C [0, 1].

16. Let X be an inner product space. Then the orthogonal complement of {0} is :

(A) X.

(B) $\{0\}$.

(C) $X \setminus \{0\}$.

(D) $X^\perp$.

17. Let φ be the bounded linear functional on $\mathbb{R}^2$ defined by $\varphi(x, y) = 2x$. Then the unique element of $\mathbb{R}^2$ representing φ given by the Riesz representation theorem is :

(A) (0, 1).

(B) (2, 0).

(C) (1, 0).

(D) (0, 2).

18. Let H be a Hilbert space over $\mathbb{R}$ and $x, y \in H$, be such that $\|x\| = 4, \|y\| = 3,$ and $\|x - y\| = 3$. Them $\langle x, y \rangle$ equals :

(A) 6.

(B) 8.

(C) 10.

(D) None of these.

19. Which of the following is Cauchy-Schwartz inequality ?

(A) $|\langle x, y\rangle| \leq \langle x, x\rangle^{1/2} \cdot \langle y, y\rangle^{1/2}$.

(B) $|\langle x, y\rangle| \geq \langle x, x\rangle^{1/2} \cdot \langle y, y\rangle^{1/2}$.

(C) $|\langle x, y\rangle| \leq \langle x, y\rangle^{1/2} \cdot \langle y, x\rangle^{1/2}$.

(D) $|\langle x, y\rangle| \leq \langle x, x\rangle \cdot \langle y, y\rangle$.

20. Which of the following is true ?

(A) If A, B are invertible linear operators on X, then A + B is invertible.

(B) If A, B are invertible linear operators on X, then A – B is invertible.

(C) If A, B are invertible linear operators on X, then AB is invertible.

(D) If A is invertible linear operator on X, and k is any scalar, then kA is invertible.

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D 11677 **(Pages : 3)** **Name..................................**

Reg. No...............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 13—FUNCTIONAL ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Consider the subspace E_1 of a linear space E. Prove that the dimension of E/E_1 is n if and only if there exists linearly independent vectors $x_1, x_2 \dots x_n$ linearly independent vectors relative to E_1 such that every vector of E can be uniquely expressed as a sum of their linear combination and a unique vector $y \in E_1$.

2. Is C [0, 1] a normed space ? Justify your answer.

3. State Holder's inequality and derive Cauchy Schwartz inequality from the same.

4. Show that inner product $\langle x, y \rangle$ is a continuous function with respect to both variables .

5. Prove that any two separable infinite dimensional Hilbert spaces H_1, H_2 are isometrically equivalent.

6. Let $f \in E^H / \{0\}$. Show that *codim ker f* = 1.

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7. State Arzela's theorem.

8. Show that there exists a linear functional on a normed space X that distinguishes distinct elements of X.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question carries weightage 2.

Unit 1

9. Show that norm is a continuous function.

10. Is a quotient space a normed space ? Justify your answer.

11. Show that the kernel for a seminorm p is a subspace of a linear space on which it is defined. Also show that $p\ (x + y)$ is independent of, where y is an element of the subspace.

Unit 2

12. State Bessel's inequality and use it to show that a complete orthonormal system in a Hilbert space H is a basis in H.

13. Prove that f is a bounded functional on a normed space X if and only if f is continuous.

14. If E is a closed subspace of a Hilbert space H and codim E = 1 then prove that $E^{\perp}$ is a one dimensional subspace.

Unit 3

15. Show that l_1 can be identified as the dual space of c_0.

16. Prove that the dual space of any normed space is complete.

17. Let X, Y be any *two* Banach spaces .Prove that for a linear operator $A : X \to Y$ implies $A^* : X^* \to Y^*$ is compact.

(6 × 2 = 12 weightage)

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Part C

Answer **two** *questions.*
Each question carries weightage 5.

18. Let E be a normed space. Show that there exists a complete normed space $\hat{E}$ and linear operator $T : E \to \hat{E}$ such that :

    (i) $\|T(x)\| = \|x\|$.

    (ii) Im (T) is a dense set in $\hat{E}$.

19. State and prove a necessary condition for a Hilbert space to have an orthonormal basis.

20. (a) Consider $f \in E^{*} / \{0\}$. Prove that :

    (i) $codim\ ker f = 1$.

    (ii) $f, g \in E^{\#} / \{0\}$ and $\ker f = \ker g$ *then there exists* $\lambda \neq 0$ such that $\lambda f = g$.

    (iii) If L is a closed subspace of E and $codim$ L = 1 then there exists $f \in E^{\#}$ such that $\ker f = 1$.

    (b) Illustrate with an example the concept of non-separable Hilbert space.

21. (a) Discuss the compactness of the integral operator in L_2.

    (b) State and prove necessary and sufficient condition for a set to be relatively compact in a normed space.

(2 × 5 = 10 weightage)

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D 11676-A **(Pages : 4)** **Name.....................................**

Reg. No..................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.

2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.

3. Write the Name, Register Number and the Date of Examination in the space provided.

4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.

5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

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MTH 3C 12—COMPLEX ANALYSIS

Multiple Choice Questions :

1. For the complex number z and ∞ in $\mathbb{C}_\infty$ the distance from z to ∞, denoted by $d(z,\infty)$, is given by ————.

(A) $d(z,\infty)=\frac{2}{\sqrt{|z|^2-1}}$.

(B) $d(z,\infty)=\frac{1}{\sqrt{|z|^2+1}}$.

(C) $d(z,\infty)=\frac{2}{\sqrt{|z|^2+1}}$.

(D) None of the above options.

2. A power series in powers of $z-z_0$ is an infinite series of the form :

(A) $\sum_{n=0}^{\infty} a_n (z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + ...$

(B) $\sum_{n=-1}^{\infty} a_n (z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + ...$

(C) $\sum_{n=-2}^{\infty} a_n (z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + ...$

(D) $\sum_{n=-3}^{\infty} a_n (z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + ...$

3. If G is open and connected and $f : G \to \mathbb{C}$ is differentiable with $f'(z)=0$ for all z in G, then ————.

(A) $f(z)=1$ for all z in G.

(B) $f(z)=0$ for all z in G.

(C) f is constant.

(D) None of the above options.

4. A function f is periodic with period c if ————.

(A) $f(z+c)=f(z)$ for all z in $\mathbb{C}$.

(B) $f(z+c)=f(z)$ for some z in $\mathbb{C}$.

(C) $f(z+c)=k$ (where k is constant) for all z in $\mathbb{C}$.

(D) None of the above options.

5. The derivative of a branch of the logarithm function is ————.

(A) z.

(B) z^{-2}.

(C) z^{-1}.

(D) None of the above options.

6. Which one of the following is an inversion ?

(A) $S(z)=z+a$ for some complex number a.

(B) $S(z)=az$ for some complex number $a \neq 0$.

(C) $S(z)=e^{i\theta}z$.

(D) $S(z)=\frac{1}{z}$.

7. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by :

$$S(z)=\frac{\frac{z-z_3}{z-z_4}}{\frac{z_2-z_3}{z_2-z_4}} \text{ if } z_2, z_3, z_4 \in \mathbb{C}_\infty.$$

Then $S(z_2)=$ ————.

(A) 1.

(B) 0.

(C) -1.

(D) None of the above options.

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8. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by :

$$S(z) = \frac{\dfrac{z - z_3}{z - z_4}}{\dfrac{z_2 - z_3}{z_2 - z_4}} \text{ if } z_2, z_3, z_4 \in \mathbb{C}_\infty.$$

Then $S(z_3) =$ ————————.

(A) 1. (B) 0.

(C) – 1. (D) None of the above options.

9. Define $\gamma : [0, 2\pi] \to \mathbb{C}$ by $\gamma(t) = e^{it}$. Then ————————.

(A) $\int_\gamma \frac{1}{z} dz = 0.$ (B) $\int_\gamma \frac{1}{z} dz = -2\pi i.$

(C) $\int_\gamma \frac{1}{z} dz = 2\pi i.$ (D) $\int_\gamma \frac{1}{z} dz = \pi i.$

10. Define $\gamma : [0, 2\pi] \to \mathbb{C}$ by $\gamma(t) = e^{it}$. Then ————————.

(A) $\int_\gamma z^2\, dz = -2\pi i.$ (B) $\int_\gamma z^2\, dz = 2\pi i.$

(C) $\int_\gamma z^2\, dz = \pi i.$ (D) $\int_\gamma z^2\, dz = 0.$

11. Define $\gamma : [0, 2\pi] \to \mathbb{C}$ by $\gamma(t) = e^{it}$. Then ————————.

(A) $\int_\gamma z^3\, dz = -2\pi i.$ (B) $\int_\gamma z^3\, dz = 2\pi i.$

(C) $\int_\gamma z^3\, dz = 0.$ (D) $\int_\gamma z^3\, dz = \pi i.$

12. Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If $\gamma_1, \gamma_2, ..., \gamma_m$ are closed rectifiable curves in G such that

$n(\gamma_1 ; w) + n(\gamma_2 ; w) + ... + n(\gamma_n ; w) = 0$ for all $w \in \mathbb{C} \sim G$,

then for $a \in G \sim \bigcup_{k=1}^{m} \{\gamma_k\}$ ————————.

(A) $\sum_{k=1}^{m} n(\gamma_k ; a) = \sum_{k=1}^{m} \frac{1}{2\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$ (B) $f(a) \sum_{k=1}^{m} n(\gamma_k ; a) = \sum_{k=1}^{m} \frac{1}{2\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

(C) $f(a) \sum_{k=1}^{m} n(\gamma_k ; a) = \sum_{k=1}^{m} \frac{1}{\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$ (D) $f(a) \sum_{k=1}^{m} n(\gamma_k ; a) = \sum_{k=1}^{m} \frac{1}{2\pi} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

13. Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic . If $\gamma_1, \gamma_2, ..., \gamma_m$ are closed rectifiable curves in G such that

$n(\gamma_1 ; w) + n(\gamma_2 ; w) + ... + n(\gamma_n ; w) = 0$ for all $w \in \mathbb{C} \sim G$,

then ————————.

(A) $\sum_{k=1}^{m} \int_{\gamma_k} f(z)\, dz = 2\pi i.$ (B) $\sum_{k=1}^{m} \int_{\gamma_k} f(z)\, dz = 2\pi.$

(C) $\sum_{k=1}^{m} \int_{\gamma_k} f(z)\, dz = 0.$ (D) $\sum_{k=1}^{m} \int_{\gamma_k} f(z)\, dz = 1.$

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14. A set G is a-star shaped if ——————.

(A) There are some points z in G such that the line segment with end points a and z lies entirely in G.

(B) For each z in G, the line segment with end points a and z lies entirely in G.

(C) Given any *two* points a and b in G, the line segment joining a and b lies entirely in G.

(D) None of the above options.

15. If G is simply connected then for every closed rectifiable curve and every analytic function f, $\int_\gamma f =$ ——————.

(A) 0. (B) 1.

(C) π. (D) 2π.

16. If γ is the circle $|z| = 2$, then $\int_\gamma \frac{2z}{z^2 + z + 1}\, dz =$ ——————.

(A) 0. (B) 1.

(C) π. (D) 2π.

17. The function $f(z) = e^z$ ——————.

(A) Has 0 as the only one singular point.

(B) Has 1 as the only one singular point.

(C) Has 2π as the only one singular point.

(D) Has no singular point.

18. Let $z = a$ be an isolated singularity of f and let

$$f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$$

be its Laurent Expansion in ann $(a\,; 0, \mathrm{R})$. Then ——————.

(A) $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \geq -1$.

(B) $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \leq -1$.

(C) $z = a$ is an essential singular point if and only if $a_{-m} = 0$ for $n \leq -(m+1)$.

(D) $z = a$ is a removable singularity if and only if $a_{-m} = 0$ for $n \leq -(m+1)$.

19. Let f have an isolated singularity at $z = a$ and let

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$$

be its Laurent Expansion about $z = a$. The residue of f at $z = a$ *is* ——————.

(A) The co-efficient a_{-2}. (B) The co-efficient a_0.

(C) The co-efficient a_1. (D) The co-efficient a_{-1}.

20. Suppose that f has a plot at $z = a$. Then $\lim_{z \to a} |f(z)| =$ ——————.

(A) ∞. (B) 0.

(C) 1. (D) $-\infty$.

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D 11676 (Pages : 3) Name.......................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n, a \in \mathbb{C}$.
2. What is Mobius transformation ? Is the mapping $T(z) = \bar{z}$ a Mobius transformation. Justify your claim.
3. Find the image of the following points in the complex plane 0, 1 + i, 3 + 2i on the unit sphere.
4. Show that lim $n^{\frac{1}{n}} = 1$.
5. If $\sum a_n$ converges absolutely then prove that $\sum a_n$.
6. Find the image of the lines $x = \alpha$ under the mapping cos z .

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7. Determine the type of singularity of $f(z)=\frac{\cos z}{z}$ at $z = 0$.

8. Find the residue of $f(z) = \tan z$ at $z=\frac{\pi}{2}$.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing two from each unit.*
Each question has weightage 2.

Unit I

9. Define $\gamma:[0,2\pi]\to C$ by $\gamma(t)=\exp(int)$ where n is some integer (positive, negative or zero). Show that $\int_{\gamma}\frac{1}{z}\,dz=2\pi in$.

10. Prove that there is no branch of the logarithm defined on G = C – {0}.

11. Prove that a Mobius transformation carries circles into circles.

Unit II

12. Show that the Integral Formula follows from Cauchy's Theorem.

13. If $\gamma:[0,1]\to C$ is a closed rectifiable curve in G such that $\gamma \sim 0$ then prove that $n(\gamma:w)$ for all w in C – G.

14. Find $\int_{\gamma} z^{\frac{-1}{2}}\,dz$ where γ is the upper half of the unit circle from +1 to – 1.

Unit III

15. If γ is piecewise differentiable and $f:[a,b]\to C$ is continuous then prove that $\int_a^b f dr=\int_a^b f(t)\gamma'(t)\,dt$.

16. If G is a region and suppose that $f: G\to C$ is analytic and $a\in G$ such that $|f(a)|\le|f(z)|$ for all z in G. Show that either $f(a) = 0$ or f is constant.

17. If γ is a closed rectifiable curve in G such that $\gamma\sim 0$ then $n(\gamma\,;w)=0$ for all w in C – G.

(6 × 2 = 12 weightage)

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Part C

Answer **two** *questions.*
Each question has weightage 5.

18. Let $f(z) = \sum_{n=0}^{\infty} a_n (z = a)^n$ have radius of convergence R > 0. Then :

    (a) For each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R.

    (b) The function f is infinitely differentiable on B (a,R) and, furthermore, $f^k(z)$ is given by $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)a_n(z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| < \text{R}$.

    (c) For $k \geq 1, a_n = \frac{1}{n!}f^n(a)$.

19. State and prove open mapping theorem.

20. State and prove Goursat's Theorem.

21. (a) Evaluate the integral $\int_{\gamma} \frac{e^z - e^{-z}}{z^n} dz$ where n is a positive integer and $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$.

    (b) Prove the following Minimum Principle. If f is a non-constant analytic function on a bounded open set G and is continuous on G^{-}, then either f has a zero in G or $|f|$ assumes its minimum value on ∂G.

(2 × 5 = 10 weightage)

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D 11675-A (Pages : 4) Name.....................................

Reg. No..................................

THIRD SEMESTER P.G. (CBCSS) DEGREE [REGULAR] EXAMINATION NOVEMBER 2020

(SDE)

Mathematics

MTH 3C 11—MULTIVARIATE CALCULUS AND GEOMETRY

(2019 Admissions)

	DD		MM		YEAR				
Date of Examination :	☐	☐	☐	☐	☐	☐	☐	☐	FN/AN

Time : 15 Minutes **Total No. of Questions : 20**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. Immediately after the commencement of the examination, the candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Write the Name, Register Number and the Date of Examination in the space provided.
4. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer book.
5. **Candidate should handover this Question paper to the invigilator after 15 minutes and before receiving the question paper for Part B Examination.**

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MTH 3C 11—MULTIVARIATE CALCULUS AND GEOMETRY

Multiple Choice Questions :

1. Let r be a positive integer. If a vector space X is spanned by a set of r vectors then :

(A) $\dim X \geq r$.

(B) $\dim X = r$.

(C) $\dim X \leq r$.

(D) $\dim X > r$.

2. Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $A(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$ then

(A) $[A'(1,0)] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

(B) $[A'(1,0)] = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

(C) $[A'(1,0)] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

(D) $[A'(1,0)] = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

3. A parametrization of the parabola $y = x^2 ; x \in \mathbb{R}$ is :

(A) $\gamma(t) = (t, t^2) ; t \in \mathbb{R}$.

(B) $\gamma(t) = (t, 2t) ; t \in \mathbb{R}$.

(C) $\gamma(t) = (t^2, t^4) ; t \in \mathbb{R}$.

(D) $\gamma(t) = (2t, 2t^2) ; t \in \mathbb{R}$.

4. Parametrization of the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

(A) $\gamma(t) = (3\cos t, 2\sin t)$.

(B) $\gamma(t) = (2\cos t, 3\sin t)$.

(C) $\gamma(t) = (2\cos t, 2\sin t)$.

(D) $\gamma(t) = (3\cos t, 3\sin t)$.

5. Speed of the curve $\gamma(t) = \left(\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t\right)$ is :

(A) $\sqrt{2t}$.

(B) 0.

(C) $2\sqrt{2}$.

(D) 1.

6. Speed of the curve $\gamma(t) = (t, t^2, t^3)$ is :

(A) 1.

(B) $\sqrt{1 + 4t^2 + 9t^4}$.

(C) 0.

(D) $\sqrt{2t}$.

7. Which of the following curve is not regular ?

(A) $\gamma(t) = \left(\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t\right)$.

(B) $\gamma(t) = (e^t, t^2)$.

(C) $\gamma(t) = (t, \cosh t)$.

(D) $\gamma(t) = (t^3 . t^6)$.

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8. Curvature of the curve $\gamma(t)=\left(\frac{4}{5}\cos t, 1-\sin t, -\frac{3}{5}\cos t\right)$ is :

(A) $\kappa = 1.$ (B) $\kappa = 0.$

(C) $\kappa = \cos t.$ (D) $\kappa = \frac{12}{5}\sin t \cos t.$

9. Curvature of the circle of radius a is :

(A) $a.$ (B) $\frac{1}{a^2}.$

(C) $\frac{1}{a}.$ (D) $a^2.$

10. The torsion of the circular helix $\gamma(\theta)=(a\cos\theta, a\sin\theta, b\theta)$ is :

(A) $\frac{b}{a^2+b^2}.$ (B) $\frac{a}{a^2+b^2}.$

(C) $\frac{1}{a^2+b^2}.$ (D) $\frac{ab}{a^2+b^2}.$

11. A parametrization of the circular cylinder of radius 1 and axis the z-axis is :

(A) $\sigma(u,v)=(v, \cos u, \sin u).$ (B) $\sigma(u,v)=(\cos u, v, \sin u).$

(C) $\sigma(u,v)=(\cos u, \sin u, v).$ (D) $\sigma(u,v)=(\cos u, \sin u, \cos v).$

12. First fundamental form of the surface patch $\sigma(u,v)=\left(u-v, u+v, u^2+v^2\right)$ is :

(A) $\left(2+4u^2\right)du^2+8uv\,dudv+\left(2+4v^2\right)dv^2.$

(B) $du^2+dv^2.$

(C) $\left(1+4u^2\right)du^2+8uv\,dudv+\left(1+4v^2\right)dv^2.$

(D) $\left(v^2+u^2\right)\left(du^2+dv^2\right).$

13. First fundamental form of the surafce patch $\sigma(u,v)=\left(u, v, v^2+u^2\right)$ is :

(A) $du^2+dv^2.$ (B) $\left(1+4u^2\right)du^2+8uv\,dudv+\left(1+4v^2\right)dv^2.$

(C) $\left(v^2+u^2\right)\left(du^2+dv^2\right).$ (D) $\left(2+4u^2\right)du^2+8uv\,dudv+\left(2+4v^2\right)dv^2.$

14. $\| \sigma_u \times \sigma_v \| =$

(A) $\left(FG-E^2\right)^{1/2}.$ (B) $\left(EF-G^2\right)^{1/2}.$

(C) $\left(EG-F^2\right)^{1/2}.$ (D) $\left(EF-G^2\right).$

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15. Let p be a point of a surface S, let $\sigma(u,v)$ be a surface patch with standard unit normal $N(u,v)$ and let $L du^2 + 2M\,du dv + N\,dv^2$ be the second fundamental form of $\boldsymbol{\sigma}$ then :

    (A) $N_u \cdot \sigma_v = LN - M^2$. (B) $N_u \cdot \sigma_v = -M$.

    (C) $N_u \cdot \sigma_v = -N$. (D) $N_u \cdot \sigma_v = -L$.

16. Let p be a point of a surface S, let $\sigma(u,v)$ be a surface patch with standard unit normal $N(u,v)$ and let $L du^2 + 2M\,du dv + N\,dv^2$ be the second fundamental form of $\boldsymbol{\sigma}$ then :

    (A) $N_v \cdot \sigma_u = -N$. (B) $N_v \cdot \sigma_u = -L$.

    (C) $N_v \cdot \sigma_u = -M$. (D) $N_v \cdot \sigma_u = LN - M^2$.

17. Let p be a point of a surface S, let $\sigma(u,v)$ be a surface patch with standard unit normal $N(u,v)$ and let $L du^2 + 2M\,du dv + N\,dv^2$ be the second fundamental form of $\boldsymbol{\sigma}$ then :

    (A) $N_v \cdot \sigma_v = -M$. (B) $N_v \cdot \sigma_v = LN - M^2$.

    (C) $N_v \cdot \sigma_v = -L$. (D) $N_v \cdot \sigma_v = -N$.

18. Let $\boldsymbol{\sigma}$ be a surface patch of an oriented surface S with first and second fundamental forms $E du^2 + 2F du dv + G dv^2$ and $L du^2 + 2M\,du dv + N\,dv^2$ respectively. Then the Gaussian curvature K of S at p is :

    (A) $K = \dfrac{LN - M^2}{EG - F^2}$. (B) $K = \dfrac{LG - 2MF + NE}{EG - F^2}$.

    (C) $K = \dfrac{LN - M^2}{2(EG - F^2)}$. (D) $K = \dfrac{LG - 2MF + NE}{2(EG - F^2)}$.

19. The Gaussian curvature of a ruled surface is :

    (A) Positive or zero. (B) Negative or zero.

    (C) 0. (D) 1.

20. The Gaussian curvature of unit sphere is :

    (A) Positive or zero. (B) 0.

    (C) Negative or zero. (D) 1.

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D 11675 (Pages : 3) Name.......................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section/Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section/sub part/ sub division may be ignored.*
4. *There will be an overall ceiling for each Section/Part that is equivalent to the maximum weightage of the Section/Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that if $A \in L(\mathbb{R}^n, \mathbb{R}^m)$, then $\| A \| < \infty$ and A is a uniformly continuous mapping of $\mathbb{R}^n$ into $\mathbb{R}^m$.

2. Show that det $[A] = 0$ if $[A]$ is $n \times n$ matrices having two equal columns.

3. Define a parametrized curve. Find the parametrization for the level curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

4. Verify whether $\sigma(u, v) = (u, v^2, v^3)$; $u, v \in \mathbb{R}$ a regular surface patch or not.

Turn over

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5. Find the equation of the tangent plane of the surface patch $\sigma(u,v)=(u,v,u^2-v^2)$ at the point $(1,1,0)$.

6. Show that $x^2+y^2=z^2$ is a smooth surfaces.

7. Calculate the first fundamental forms of the surface $\sigma(u,v)=(\cosh u, \sinh u, v)$.

8. Show that every local isometry is conformal. Give an example of a conformal map that is not a local isometry.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

Unit 1

9. Let Ω be the set of all invertible linear operator on $\mathbb{R}^n$, show that :

   (a) If $A\in\Omega, B\in L(\mathbb{R}^n)$, and $\| B-A \| , \| A^{-1} \| < 1$, then $B\in\Omega$.

   (b) Ω is an open subset of $L(\mathbb{R}^n)$, and the mapping $A\to A^{-1}$ is continuous on Ω.

10. Show that if X is a complete metric space, and if φ is a contraction of X into X, then there exist one and only one $x\in X$ such that $\varphi(x)=x$.

11. Show that a linear operator A on $\mathbb{R}^n$ is invertible if and only if $\det[A]\neq 0$.

Unit 2

12. If $\gamma(t)$ be a regular curve in $\mathbb{R}^3$, then show that its curvature is $\kappa=\dfrac{\| \ddot{\gamma}\times\dot{\gamma} \|}{\| \dot{\gamma} \|^3}$.

13. Let γ be a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Then, show that γ is a parametrization of part of a circle.

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14. Suppose that two smooth surfaces S and $\tilde{S}$ are diffeomorphic and that S is orientable. Prove that $\tilde{S}$ is orientable.

Unit 3

15. Show that any tangent developable is locally isometric to a plane.

16. Calculate the Gaussian curvature of $\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$ where $f > 0$ and $\dot{f}^2 + \dot{g}^2 = 1$.

17. Calculate the principal curvatures of the catenoid $\sigma(u, v) = (\cosh u \cos v, \cosh u \sin v, u)$.

(6 × 2 = 12 weightage)

Part C

Answer **two** *questions.*
Each question has weightage 5.

18. State and prove the Implicit function theorem.

19. Let $\gamma(s)$ and $\tilde{\gamma}(s)$ be two unit-speed curves in $\mathbb{R}^3$ with the same curvature $\kappa(s) > 0$ and the same torsion $\tau(s)$ for all *s*. Then, there is a direct isometry M of $\mathbb{R}^3$ such that $\tilde{\gamma}(s) = M(\gamma(s))$ for all *s*. Further, if *k* and *t* are smooth functions with $k > 0$ everywhere, there is a unit-speed curve in $\mathbb{R}^3$ whose curvature is *k* and whose torsion is *t*.

20. Let S and $\tilde{S}$ be surfaces and let $f : S \to \tilde{S}$ be a smooth map. Then, prove that *f* is a local diffeomorphism if and only if, for all $p \in S$, the linear map $D_p f : T_p S \to T_{f(p)} \tilde{S}$ is invertible.

21. A local diffeomorphism $f : S_1 \to S_2$ is conformal if and only if there is a function $\lambda : S_1 \to \mathbb{R}$ such that $f^* \langle v, w \rangle_p = \lambda(p) \langle v, w \rangle_p$ for all $p \in S_1$ and $v, w \in T_p S_1$.

(2 × 5 = 10 weightage)

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D 11377 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2021

(CUCSS)

Mathematics

MT 3C 15—PDE AND INTEGRAL EQUATIONS

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Find the partial differential equation of all spheres of radius r, having center in the xy plane.
2. Find the complete and singular integral of $z = px + qy + \log (pq)$.
3. Verify that the two equations $xp = yq + x$ and $x^2p + q = xz$ are compatible.
4. Find the complete integral of $9 (p^2z + q^2) = 4$.
5. Determine the integral surface of the equation $p^2x + qy - z = 0$ which containing the line $y = 1, x + z = 0$.
6. State Cauchy's problem for first order equations.
7. Write down the one dimensional wave equations and its D'Alembert's solution.
8. Show that the solution of the Neumann problem is unique up to the addition of constant.
9. Show that the heat conduction on finite rod of length with Dirichlet boundary has unique solution.
10. Show that the function $y(x) = \left(1 + x^2\right)^{-\frac{3}{2}}$ is solution of the Volterra integral equation :

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{\xi}{1+x^2} y(\xi)\, d\xi.$$

11. Define Frodholm integral equation with separable kernel and give an example for it.
12. Determine the characteristic value λ for the equation $y(x) = \lambda \int_0^1 \left(2x\xi - 4x^2\right) y(\xi)\, d\xi.$

Turn over

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13. Find the resolvent kernel of the Volterra equation with the kernel $K(x, \xi) = \dfrac{2 + \cos x}{2 + \cos \xi}$.

14. Show that the resolvent kernel satisfies the integro-differential equation :

$$\frac{\partial R(x, \xi ; \lambda)}{\partial \lambda} = \int_a^b R(x, z ; \lambda) R(z, \xi ; \lambda) \, dz.$$

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Find the general integral of the equation $\left(x^3 + 3xy^2\right) p + \left(y^3 + 3x^2 y\right) q = 2z \left(x^2 + y^2\right)$.

16. Using Charpit's method find a complete integral of a first order partial differential equation $px + 3qy = 2\left(z - x^2 q^2\right)$.

17. Show that a complete integral of the equation $f\left(\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial u}{\partial z}\right) = 0$ is $uax + by + \theta(a, b) z + c$, where a, b and c are arbitrary constants and $f(a, b, \theta) = 0$.

18. Reduce the equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ into canonical form and hence solve it.

19. Show that $v(x, y ; \alpha, \beta) = \dfrac{(x + y)\left[2xy + (\alpha - \beta)(x - y) + 2\alpha\beta\right]}{(\alpha + \beta)^3}$ is the Riemann function for the second order partial differential equation $u_{xy} + \dfrac{2}{x + y}\left(u_x + u_y\right) = 0$.

20. Evaluate the steady state temperature in a rectangular plate of length a and width b, the sides of which are kept at temperature zero, the lower end is kept at temperature $f(x)$ and the upper edge is kept insulated.

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21. Find the solution of the one-dimensional diffusion equation $ku_{xx} = u_t$ satisfying the following boundary conditions :

(i) u is bounded as $t \to \infty$.

(ii) $u_x(0, t) = 0 = u_x(a, t)$ for all t.

(iii) $u(x, 0) = x(a - x), 0 < x < a.$

22. Transform the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(l) = 0$ to an integral equation.

23. Show that the set of eigenvalues of the second iterated kernel coincide with the set of square of the eigenvalues of the given kernel.

24. Find the resolvent kernel of the integral equation $y(x) = e^{x^2} + \int_0^x e^{x^2 - \xi^2} y(\xi) \, d\xi.$

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Show that the Paffian differential equation :

$$\left(y^2 + yz\right) dx + \left(xz + z^2\right) dy + \left(y^2 - xy\right) dz = 0$$

is integrable and find the corresponding integral.

26. Determine the characteristic of the equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$.

27. Derive the linear one dimensional wave equation for transverse vibration of string and describe "domain of dependence and range of influence".

28. Show that any solution of the integral equation :

$$y(x) = \lambda \int_0^1 (1 - 3x\xi) \, y(\xi) \, d\xi + F(x)$$

can be expressed as the sum of F (x) and some linear combination of the characteristic functions.

(2 × 4 = 8 weightage)

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D 11376 (Pages : 3) Name..

Reg. No..

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2021

(CUCSS)

Mathematics

MT 3C 14—FUNCTIONAL ANALYSIS

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

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Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Show that the metric space l^∞ is not separable.

2. Show that for $p \geq 1$, the sequence space $l^p \subset l^\infty$.

3. Show by an example that a linear map on a linear space X may be continuous with respect to some norm on X, but discontinuous with respect to another norm on X.

4. State the Hahn-Banach extension theorem.

5. Is c_{00}, the space of all scalar sequences having only finitely many non-zero terms as a subspace of l^∞, a closed set ? Justify your answer.

6. State the Uniform boundedness principle.

7. State the parallelogram law in inner product spaces.

8. What is known as the Minkowski space ?

9. State the Minkowski's inequality for sequences.

10. Define an essentially bounded function on a set.

11. Show that the metric space $L^p[a, b]$ is separable for $1 \leq p < \infty$.

12. Show that the function $\| \ \|$, is uniformly continuous on a normed space X.

Turn over

13. Show that the hypothesis of completeness on the domain cannot be dropped in the Uniform boundedness principle.

14. Let $\langle , \rangle$ be an inner product on a linear space X. If $\| x_n - x \| \to 0$ and $\| y_n - y \| \to 0$, then, show that $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Give an example to show that the open mapping theorem may not hold true if the normed spaces X is not a Banach space even when Y is a Banach spaces.

16. Show that a linear map F from a normed space X to a normed space Y is a homeomorphism if there are $\alpha, \beta > 0$ such that $\beta \| x \| \le \| F(x) \| \le \alpha \| x \|$ for all $x \in X$.

17. Let X and Y be normed spaces. If Z is a closed subspace of X, then show that the quotient map Q from X to X/Z is continuous and open.

18. Let X be a normed space, Y be a closed subspace of X and $Y \neq X$. Let r be a real number such that $0 < r < 1$. Show that there exists some $x_r \in X$ such that $\| x_r \| = 1$ and $r \le dis(x_r, Y) \le 1$.

19. State and prove the Polarization identity in an inner product space.

20. Let X be an inner product spaces. Let $\{x_1, x_2, ..., x_n\}$ be an orthogonal set in X, then show that $\| x_1 + x_2 + ... + x_n \|^2 = \| x_1 \|^2 + \| x_2 \|^2 + ... + \| x_n \|^2$.

21. Give an example to show that in a non-complete metric space, the intersection of a denumerable number of dense open subsets need not be dense.

22. Let X be a normed space and E be a convex subset of X. Show that the closure $\overline{E}$ of E is convex.

23. When do you say that a linear functional is positive ? Give an example for a positive linear functional.

24. Let X be a normed space over K, and f be a non-zero linear functional on X. If E is an open subset of X, then show that $f(E)$ is an open subset of K.

(7 × 2 = 14 weightage)

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Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Let X be a metric space. Then show that :

    (i) The intersection of a finite number of dense open subsets of X is dense in X.

    (ii) If X is complete, then the intersection of a countable number of dense open subsets of X is dense in X.

26. State and prove the open mapping theorem.

27. State and prove the Gram-Schmidt orthoganalization theorem.

28. Let X and Y are normed spaces and $X \neq 0$. Then show that :

    (a) $BL(X, Y)$, the set of all bounded linear maps from X to Y is a Banach space in the operator norm if and only if Y is a Banach space.

    (b) The dual X' of every normed space X is a Banach space.

(2 × 4 = 8 weightage)

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D 11375 (Pages : 3) Name.....................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2021

(CUCSS)

Mathematics

MT 3C 13—COMPLEX ANALYSIS

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Show that if $f(z)$ is an analytic function of $z = x + iy$, then x and y can occur in $f(z)$ only in the combination $x + iy$.

2. Give an example of a function f on $\mathbb{C}$ such that f_x and f_y exist and are continuous at a point, but it is not differentiable at the same point.

3. Show that the Mobius transformations preserves cross ratios.

4. Find the linear transformations that maps $1-i, i, 0$ into $-i, 1+i, 1$ respectively.

5. Let $\gamma : [0,1] \to \mathbb{C}$ by $\gamma(t) = t + it$. Evaluate $\int_{\gamma} (1+z)\, dz$.

6. Determine the nature of the singularity of $\sin\left(\frac{1}{1-z}\right)$ at $z = 1$.

7. Find the residue of $\frac{e^z}{\sin z}$ at simple pole.

8. Piove that a bounded entire function is a constant.

9. If $P(z)$ is a non-constant polynomial in C, then prove that $P(z)$ has a root in C.

10. Develop $\tan(z)$ in powers of z upto the term containing z^4.

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Turn over

11. If $f(z)=\frac{1}{z^2-5z+6}$, give Laurent series expansion of f in the annuls region $0<|z|<2$.

12. State Rouch's theorem.

13. Show that if f is analytic is a region G and a is a point with $|f(a)|\geq|f(z)|$ for all z in G, then f is a constant function.

14. If f is an elliptic function, then f' is also an elliptic function.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. If T is a linear fractional transform such that T is not identity, then T has at most two fixed points in $\mathbb{C}$.

16. Explain symmetry with respect to a straight line.

17. Find an analytic function $f(z)=u+iv$ whose real part $u=\frac{\sin(2x)}{\cosh(2y)-\cos(2x)}$.

18. Suppose $f(z)$ is analytic in the region Ω' obtained by omitting a point a from the region Ω.

Prove that a is a removable singularity of $f(z)$ if $\lim_{z\to a}(z-a)f(z)=0$.

19. State and prove Cauchy's integral formula for derivatives.

20. State and prove Morera's theorem.

21. Define winding number. Prove that $n(\gamma,a)=n(\gamma,b)$, where a,b belongs to same region determined by the closed curve γ in $\mathbb{C}$.

22. By the method of residues, evaluate $\int_0^{\pi}\frac{1}{5+3\cos\theta}\,d\theta$.

23. How many roots of the equation $z^5+15z+1$ lie in the annular region $\frac{3}{2}<|z|<2$.

24. Show that the sum of residues of an elliptic function is zero.

(7 × 2 = 14 weightage)

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Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Let (f_n) be a sequence of analytic functions such that f_n is nowhere zero for all $n \in \mathbb{N}$. If $f_n \to f$ uniformly on every compact subset of Ω as $n \to \infty$, then prove that either f is identically zero or f is nowhere zero.
26. State and prove Argument principle.
27. Prove that any two bases of the same module are connected by a unimodular transformation.
28. Show that the Weierstrass $\wp$-function he following differential equation.

$$\left(\wp'_{a,b}(z)\right)^2 = 4\left(\wp_{a,b}(z)\right)^3 - 60\mathrm{G}_{4\wp_{a,b}}(z) - 140\mathrm{G}_6.$$

(2 × 4 = 8 weightage)

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D 11374 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2021

(CUCSS)

Mathematics

MT 3C 12—MULTIVARIABLE CALCULUS AND GEOMETRY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

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Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. State the Banach Contraction Principle.

2. Use the definion of determinant to show that if the matrix $[A]$ has two equal columns, then $det\ [A] = 0$.

3. Find the parametrisation of the level curve $y^2 - x^2 = 1$.

4. If the tangent vector of a parametrised curve is constant, then show that the image of the curve is (part of) a straigt line.

5. Find the arc-length of the logarithmic spiral $\gamma(t) = (e^t \cos t, e^t \sin t)$ starting at $\gamma(0) = (1, 0)$.

6. Show that the curve $\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{t}{\sqrt{2}}\right)$ is of unit speed.

7. Give an example to show that a level curve can have both regular and non-regular parametrisation.

8. Write down the Frenet-Serret equations.

9. Show that the curve $\gamma(t) = \left(\frac{1+t^2}{t}, t+1, \frac{1-t}{t}\right)$, is planar.

Turn over

10. Find the equation of the tangent plane of the surface patch $\sigma(u, v) = (u, v, u^2 - v^2)$ at the point $(1, 1, 0)$.

11. What is meant by a quadric ? Give an example.

12. Show that any (part of a) straight line on a surface is a geodesic.

13. Describe four different geodesics on the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$ passing through the point $(1, 0, 0)$.

14. Is it possible to define Gauss map for any orientable surface ? If so, why ?

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Suppose X is a vector space of dimension n. Prove that X has a basis and every basis consists of n vectors.

16. Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E, and there is a real number M such that $\| f'(x) \| \leq M$ for every $x \in E$. Show that $| f(b) - f(a) | \leq M | b - a |$ for all $a, b \in E$.

17. Prove that a linear operator A on R^n is invertible if and only if $det\ [A] \neq 0$.

18. Show that the curvature of the circle centered at (x_0, y_0) and of radius R is inversely proportional to its radius.

19. Let γ be a unit speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Then show that γ is (part of) a circle.

20. Calculate the Gaussian and Mean curvatures of the surface $\sigma(u, v) = (u + v, u - v, uv)$ at the point $(2, 0, 1)$.

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21. Show that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.

22. Show that if P and Q are distinct points of a circular cylinder, there are either two or infinitely many geodesics on the cylinder joining P and Q. Which pairs P, Q have the former property ?

23. Find the geodesics on a circular cylinder by solving the geodesic equations.

24. Show that Mobius band is not an orientable surface.

($7 \times 2 = 14$ weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$.

 (i) Show that its curvature is $\kappa = \dfrac{\| \ddot{\gamma} \times \dot{\gamma} \|}{\| \dot{\gamma} \|^3}$, where the $\times$ indicates the vector product and the dot denotes d/dt.

 (ii) Use the above formula to compute the curvature of the helix with axis the z-axis.

26. Let $\gamma(s)$ and $\tilde{\gamma}(s)$ be two unit-speed curves in $\mathbb{R}^3$ with the same curvature $\kappa(s) > 0$ and the same torsion $\tau(s)$ for all s. Show that

 (i) There is a rigid motion M of $\mathbb{R}^3$ such that $\tilde{\gamma}(s) = M(\gamma(s))$ for all s.

 (ii) If k and t are smooth functions wth $k > 0$ everywhere, there is a unit-speed curve in $\mathbb{R}^3$ whose curvature is k and whose tortion is t.

27. Define a surface in $\mathbb{R}^3$. Is the unit-sphere $S^2 = \left\{(x, y, z) \in \mathbb{R}^3 \,\middle|\, x^2 + y^2 + z^2 = 1\right\}$ a surface ? Justify your answer.

28. Let P be a point of a flat surface S, and assume that P is not an umbilic. Show that there is a patch of S containing P that is a ruled surface.

($2 \times 4 = 8$ weightage)

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D 11219 (Pages : 3) Name..............................

Reg. No.............................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Mathematics

MAT 3E 01—ADVANCED TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 marks.

1. Prove that a compact subset in a Hausdorff space is closed.

2. Let A, B be subsets of a space X and suppose there exists a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$. Prove that there exist disjoint open sets U, V such that $A \subset U$ and $B \subset V$.

3. Prove that every path connected space is connected.

4. Show that T_2 property is productive.

5. Prove that the intersection of any family of filters on a set is again a filter on that set.

6. Let X and Y be topological spaces, $x \in X$ and $f : X \to Y$ a function. If f is continuous at x, then show that a net S converges to x in X implies the net $f \circ S$ converges to $f(x)$ in Y.

7. Prove or disprove. Every complete metric space is compact.

8. Give an example to show that weak contraction need not imply contraction.

(8 × 2 = 16 marks)

Part B

Answer any **four** *questions.*
Each question carries 4 marks.

9. Prove that every regular, Lindeloff space is normal.

10. Show that the product topological space is completely regular if each co-ordinate space is completely regular.

Turn over

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11. Prove that a topological space is Hausdorff if the limits of all nets in it are unique.

12. Let $\mathcal{S}$ be a family of subsets of a set X. Prove that there exists a filter on X having $\mathcal{S}$ as a sub-base if and only if $\mathcal{S}$ has the finite intersection property.

13. Let A be a subset of a metric space (X, d) such that A is complete w.r.t. the metric induced on it. Prove that A is closed in X.

14. Prove that in a locally compact Hausdorff space, a subset of a first category can have no interior points.

($4 \times 4 = 16$ marks)

Part C

Answer **either** *A* **or** *B of each question.*
Each question carries 12 *marks.*

Unit I

15. A. Let A be a closed subset of a normal space X and suppose $f : \to [-1, 1]$ is a continuous function. Then show that there exists a continuous extension of f on X.

B. (a) Let A and B be compact subsets of topological spaces X and Y respectively. Let W be an open subset of X × Y containing the rectangle A × B. Show that there exist open sets U, V in X, Y respectively such that $A \subset U$, $B \subset V$ and $U \times V \subset W$.

(b) Suppose $\mathcal{D}$ is a decomposition of a space X each of whose members is compact and suppose the projection $p : X \to \mathcal{D}$ is closed. Prove that the quotient space $\mathcal{D}$ is Hausdorff of regular according as X is Hausdorff or regular.

Unit II

16. A. (a) Prove that the following statements are equivalent in a topological space X :

(i) X is locally connected.

(ii) Components of open subsets of X are open.

(iii) X has a base consisting of connected subsets.

(iv) For every $x \in X$ and every neighbourhood N of x there exists a connected open neighbourhood M of x such that $M \subset N$.

(b) Give an example of a space which is connected but not locally connected.

B. Prove that metrisability is a countably productive property.

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Unit III

17. A. (a) Define cluster point of a net and subnet of a net.

(b) Let S : D → X be a net in a topological space and let $x \in$ X. Then prove that x is a cluster point of S if and only if there exists a subnet of S which converges to x in X.

B. (a) Let $\mathcal{F}$ be a filter in a space X and $\mathcal{S}$ be the associated net in X. Prove that $\mathcal{F}$ converges to a point $x \in$ X as a filter if and only if $\mathcal{S}$ converges to x as a net.

(b) Let X be the topological product of an indexed family of spaces $\{X_i : i \in I\}$. Let $\mathcal{F}$ be a filter on X and let $x \in$ X. Then show that $\mathcal{F}$ converges to x in X if the filter $\pi_{i\#}(\mathcal{F})$ converges to $\pi_i(x)$ in X_i.

Unit IV

18. A. Define totally bounded set. Prove that a metric space is compact if and only if it is complete and totally bounded.

B. When will you say that a topological space is an absolute G_δ. Prove that a topological space is metrically topologically complete if and only if it is an absolute G_δ set.

(4 × 12 = 48 marks)

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11. Show by an example that, a linear map on a linear space X may be continuous with respect to some norm on X but discontinuous with respect to another norm on X.

12. Consider $Z = \{(x(1), x(2)) \in X : x(1) = x(2)\}$, and define $g \in Z'$ by $g(x(1), x(2)) = x(1)$. Find any two Hahn Banach extensions of g.

13. Let X be a Banach space. Y be a normed space and (F_n) be a sequence in BL (X, Y) such that the sequence $(F_n(x))$ converges in Y for every $x \in X$. For $x \in X$, define $F(x) = \lim_{n\to\infty} F_n(x)$. Prove that $(F_n(x))$ converges to $F(x)$ uniformly for all x in a totally bounded subset E of X.

14. X be a normed space and $P : X \to X$ be a projection. Show that P is a closed map if and only if the subspaces R(P) and Z(P) are closed in X.

(4 × 4 = 16 marks)

Part C

Answer either A **or** *B of each question.*
Each question carries 12 *marks.*

15. A. (a) State and prove Baire's theorem for metric spaces.

(b) Show that the metric space l^∞ is not separable.

B. (a) Consider a measurable subset $E = [a, b]$ of $\mathbb{R}$ and $1 \le p \le \infty$. Show that the set of all step functions on E is dense in $L^p([a,b])$.

(b) Let $\| \ \|_j$ be a norm on a linear space X_j, $j = 1, 2, \ldots, m$. Fix p such that $1 \le p < \infty$. For $x = (x(1), \ldots, x(m))$ in the product space $X = X_1 \times X_2 \times \ldots \times X_m$, let $\|x\|_p = \left(\|x(1)\|_1^p + \ldots + \|x(m)\|_m^p\right)^{\frac{1}{p}}$. Prove that $\| \ \|_p$ is a norm on X.

16. A. (a) Show that a linear map F from a normed space X to a normed space Y is a homeomorphism if and only if there are $\alpha, \beta > 0$ such that $\beta\|x\| \le \|F(x)\| \le \alpha\|x\|$ for all $x \in X$.

(b) There exists a discontinuous linear map on an infinite dimensional normed linear space X. Prove or disprove.

B. (a) Let $M = (K_{ij})$ be an infinite matrix with scalar entries such that $\sup\left\{\sum_{i=1}^{\infty}|K_{ij}| : j = 1, 2, 3\ldots\right\} < \infty$. For $x \in l^1$, let $M(x) \in l^1$ be defined by $(Mx)(i) = \sum_{j=1}^{\infty} K_{ij}x(j)$. Show that M is a continuous linear map and find its norm.

(b) State and prove Hahn Banach separation theorem.

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17. A. (a) Let X and Y be normed space and $X \neq \{0\}$. Prove that BL (X, Y) is a Banach space in the operator norm if and only if Y is a Banach space.

(b) State and prove Hahn Banach extension theorem.

B. (a) Show that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

(b) If Y is a proper dense subspace of a Banach space X, then prove that Y is not a Banach space in the induced norm.

18. A. State and prove closed graph theorem.

B. (a) Let X and Y be normed spaces and $F : X \to Y$ is a linear map. Then prove that F is an open map if and only if there exists some $\gamma > 0$ such that for every $y \in Y$, there is some $x \in X$ with $F(x) = y$ and $\|x\| \leq \gamma \|y\|$.

(b) Give an example to show that open mapping theorem may not hold if the normed spaces X and/or Y are not complete.

($4 \times 12 = 48$ marks)

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D 11216 (Pages : 2) Name..............................

Reg. No..............................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Mathematics

MAT 3C 11—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 marks.

1. Define Complex exponential function and where it is defined.
2. Define a path in a region G subset of C and give an example of a path joining $1 + i1$ and $0 + i0$.
3. What are the zeros of $\cos\left(\frac{1+z}{1-z}\right), |z|<1$?
4. Evaluate $\int_{|z|=1} \frac{\sin(z)}{z} dz$.
5. When we say that two closed rectifiable curves are homotopic ?
6. Evaluate $\int_\gamma \frac{2z+1}{z^2+z+1} dz$, where γ is the curve $|z|=2$.
7. State Schwarz's lemma.
8. State and prove Maximum modulus principle (First version)

(8 × 2 = 16 marks)

Part B

Answer any **four** *questions.*
Each question carries 4 marks.

9. Let S be the unit sphere in R^3 centred at origin. Then define a bijection between the extended complex plane C_∞ and the sphere S and verify.
10. Define branch of logarithm and give a branch of $\log z$. Also show that it is analytic.
11. Show that if γ is a piecewise smooth and $f : [a\ b] \to C$ continuous, then $\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t)\, dt$.

Turn over

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12. If G is simply connected and $f : G \to C$ is analytic in G, $\to$ then show that f has a primitive in G.
13. State and prove open mapping theorem of analytic functions.
14. State and prove Argument principle.

(4 × 4 = 16 marks)

Part C

Answer either A **or** *B of each of the four questions.*
Each question carries 12 marks.

Unit I

15. A. (i) Prove : If G is open and connected and $f : G \to C$ is differentiable with $f'(z) = 0$ for all z in G, then f is constant.

(ii) Prove : Let G be either the whole plane C or some open disk. If $u : G \to R$ is a harmonic function, then u has a harmonic conjugate.

B. (i) Show that Mobius transformation is a bijection and it is the composition of translation, dilation and modulation.

(ii) Define cross ratio and show that cross ratio of four points is real number if and only if all four points lies on a circle.

Unit II

16. A. Let $f : G \to C$ be analytic and suppose $\overline{B(a,r)}$ subset of G. If $\gamma(t) = a + re^{it}, 0 \le t \le 2\pi$.

Then show that $f(z) = \frac{1}{2\pi} \int_\gamma \frac{f(w)}{w - z} dw$ for $|z - a| < r$.

B. Let γ be a closed rectifiable curve in C. Then show that $n(\gamma, a)$ is constant for a belong to the component of $G = C - \{\gamma\}$ and $n(\gamma, a) = 0$ for a belong to the unbounded component of G.

Unit III

17. A. State and prove Morera's theorem.

B. Let G be an open set and $f : G \to C$ be a differentiable function ; then f is analytic on G.

Unit IV

18. A. If f has an isolated singularity at a then the point $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z - a) f(z) = 0$.

B. Show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

(4 × 12 = 48 marks)

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D 11215 (Pages : 2) Name.............................

Reg. No..........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Mathematics

MAT 3E 01—ADVANCED TOPOLOGY

(2017 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *the questions.*
Each question carries 2 marks.

1. Prove that a compact subset of a Hausdorff space is closed.
2. Define Urysohn function. State the result that guarantees the existence of such a function.
3. Define path-connected topological space. Verify whether the real line (in the usual topology) is path-connected or not.
4. When do we say that a topological property is productive ? Is normality is productive property ? Justify your claim.
5. Prove that if a subset A of a topological space is closed then limits of nets in A are in A.
6. Define filter and ultrafilter. Prove that every filter is contained in an ultrafilter.
7. Prove that compact metric space is totally bounded.
8. Is it true that, in general a complete metric space is compact ? Justify your claim.

(8 × 2 = 16 marks)

Part B

Answer any **four** *questions.*
Each question carries 4 marks.

9. Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.
10. Prove that every compact Hausdorff space is T_4.
11. Prove that a product a topological spaces is path-connected if and only if each co-ordinate space is path-connected.
12. Prove that second countability is preserved under continuous open functions.

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Turn over

13. Prove that if limits of all nets in a topological space are unique, then the space is Hausdorff.

14. Define confinal subset of a directed set. Let F be a confinal subset of a directed set (D, $\geq$) in a topological space X. Then prove that if a net S : D $\rightarrow$ X converges to a point $x \in$ X, then so does its restriction S/F : F $\rightarrow$ X.

(4 × 4 = 16 marks)

Part C

Answer either A **or** *B part of the following questions.*
Each question carries 12 marks.

15. A. (a) Let X be a Hausdorff space, $x \in$ X and F a compact subset of X not containing x. Then prove that there exists open sets U, V such that $x \in$ U, F $\subset$ V and U $\cap$ V = ϕ.

(b) Prove that regular, second countable space is normal.

B. (a) Let A, B be subsets of a space X and suppose there exists a continuous function $f : X \rightarrow [0, 1]$, such that $f(x) = 0$ for all $x \in$ A and $f(x) = 1$ for all $x \in$ B. Then prove that there exist disjoint open sets U, V such that A $\subset$ U and B $\subset$ V.

(b) Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued functin on A has a continuous extension to X. Prove that X is normal.

16. A. (a) Prove that a space X is locally connected at a point $x \in$ X if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.

(b) Prove that every open subset of the real line (in the usual topology) can be expressed as the union of mutually disjoint open intervals.

B. (a) Prove that a product of space is connected if and only if each space is connected.

(b) Prove that every subspace of the Hilbert cube is a second countable metric space.

17. A. (a) Prove that in an indiscrete space, every net converges to every point of the space.

(b) Prove that a subset B of a space X is open if and only if no net in the complement X – B can converges to a point in B.

B. (a) Prove that the intersection of any family of filters on a set is again a filter on that set.

(b) Prove that any family which does not contain the empty set and which is closed under finite intersections is a base for a unique filter.

18. A. (a) Prove that every compact metric space is complete.

(b) Prove that an open subspace of a metrically topologically complete space is metrically topologically complete.

B. Prove that a topological space is metrically topologically complete if and only if it is an absolute G_δ.

(4 × 12 = 48 marks)

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D 11214 (Pages : 3) Name..............................

Reg. No...........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Mathematics

MAT 3C 12—FUNCTIONAL ANALYSIS

(2017 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 2 marks.

1. Define a Cauchy sequence in a metric space and show that a bounded sequence in a metric space need not be a Cauchy sequence.
2. Let X be a normed space. Show that if E_1 is open in X and $E_1 \subset X$, then $E_1 + E_2$ is open in X.
3. Let F be a linear map from a normed space X to a normed space Y and $\beta\|x\| \leq \|F(x)\| \leq \alpha\|x\|$ for some $\alpha, \beta > 0$ and all $x \in X$. Show that F is a homeomorphism.
4. Let X be the normed linear space K^n with norm $\| \ \|_\infty$ and let an $n \times n$ diagonal matrix diag $(k_1, \ldots, k_n)$ define a linear map $M : K^n \to K^n$. Determine $\|M\|$.
5. Let X be a normed space and let $\{a_1, \ldots, a_m\}$ be a linearly independent set in X. Show that there are $f_1, f_2, \ldots, f_m$ in X′ such that $f_j(a_i) = \delta_{ij}, i, j = 1, 2, \ldots, m$.
6. Show that the linear space c_{00} cannot be a Banach space in any norm.
7. Let X, Y and Z be metric spaces. Show that if F : X → Y is continuous and G : Y → Z is closed then GoF : X → Z is closed.
8. Let X and Y be Banach spaces and let F ε BL (X, Y) be bijective. Show that F^{-1} ε BL (Y, X).

(8 × 2 = 16 marks)

Part B

Answer any **four** *questions.*
Each question carries 4 marks.

9. Show that l^∞ is not separable.
10. Let $\| \ \|$ and $\| \ \|'$ be norms on a linear space X. Show that the two norms are equivalent iff there are $\alpha > 0$ and $\beta > 0$ such that $\beta\|x\| \leq \|x\|' \leq \alpha\|x\|$ for all x ε X.

Turn over

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11. Let $X = c_{00}$ with the norm $\| \|_1$ and let $f : X \to K$ be defined by $f(x) = \sum_{j=1}^{\infty} \frac{x(j)}{j}$ for $x \varepsilon X$. Show that f in a continuous linear functional and find $\|f\|$.

12. Let Y be a subspace of a normed space X and $a \varepsilon X$ but $a \notin \overline{Y}$. Show that there is some $f \varepsilon X'$ such that $f / Y = 0, f(a) = dist(a, \overline{Y})$ and $\|f\| = 1$.

13. Let X and Y be normed spaces and let $F : X \to Y$ be a linear map such that the subspace Z(F) is closed in X. Define $\tilde{F} : \frac{X}{Z(F)} \to Y$ by $\tilde{F}(x + Z(F)) = F(x)$ for all $x \varepsilon X$. Show that F is an open map iff $\tilde{F}$ is an open map.

14. Let X denote the sequence space l^2. Let $\| \|'$ be a complete norm on X such that if $\|x_n - x\|' \to 0$ then $x_n^{(j)} \to x(j)$ for every $j = 1, 2, \ldots$. Show that $\| \|'$ is equivalent to the norm $\| \|_2$ on X.

(4 × 4 = 16 marks)

Part C

Answer either A **or** *B part of the following questions.*
Each question carries 12 *marks.*

15. A. (a) Show that the intersection of a finite number of dense open subsets of a metric space X is dense in X.

(b) Let E be a measurable subset of R and $m(E) < \infty$. Show that if $1 \le p < \infty$, then the set of all bounded continuous functions on E is dense in L^p (E).

B. Let Y be a closed subspace of a normed space X for $x + Y$ in X/Y, let $\|\|x + Y\|\| = \inf \{\|x + y\| : y \varepsilon Y\}$.

(i) Show that $\|\| \ \|\|$ is a norm on X/Y.

(ii) Show that a sequence $(x_n + Y)$ converges to x + Y in X/Y iff there is a sequence (y_n) in Y such that $(x_n + y_n)$ converges to x in X.

16. A. (a) Let X and Y be normed spaces and F : X → Y be a linear map. Show that F is continuous on X iff $\|F(x)\| \le \alpha \|x\|$ for all $x \varepsilon X$ and some $\alpha > 0$.

(b) Show that a linear functional f on a normed linear space X is continuous iff the zero space Z (f) is closed in X.

B. (a) State and prove Hahn-Banach separation theorem.

(b) Let E be a non-empty convex subset of a normed space X over K. Show that if $E^\circ \ne \phi$ and b belongs to the boundary of E in X, then there is a nonzero $f \varepsilon X'$ such

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17. A. Let X be a normed space. Show that for every sub-space Y of X and every $g \varepsilon Y'$, there is a unique Hahn-Banach extension of g to X iff X′ is strictly convex.

B. (a) Show that a normed space X is a Banach space iff every absolutely summable series of elements in X is summable in X.

(b) Let X be a normed space and Y be a Banach space. Let X_0 be a dense subspace of X and $F_0 \varepsilon BL(X_0, Y)$. Show that there is a unique $F \varepsilon BL(X, Y)$ such that $F/X_0 = F_0$ and $\|F\| = \|F_0\|$.

18. A. (a) Let E be a subset of a normed space X. Show that E is bounded in X iff f (E) is bounded in k for every $f \varepsilon X'$.

(b) Let X and Y be normed spaces and $F : X \to Y$ be linear. Show that F is an open map iff there exists some $r > 0$ such that for every $y \varepsilon Y$, there is some $x \varepsilon X$ with $F(x) = y$ and $\|x\| \le \gamma \|y\|$.

B. Show that there are scalars $k_n \varepsilon K, n = 0, \pm 1, \pm 2, \ldots$ such that $k_n \to 0$ as $n \to \pm\infty$, but there is no $x \varepsilon L^1([-\pi, \pi])$ such that $\hat{x}(n) = k_n$ for all $n = 0, \pm 1, \pm 2, \ldots$, where $\hat{x}(n)$ is the nth Fourier coefficient of x.

(4 × 12 = 48 marks)

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