C 23357-A (Pages : 4) Name.....................................

Reg. No.................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Mathematics

MTH 2C 10—OPERATIONS RESEARCH

(2020 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
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MTH 2C 10—OPERATIONS RESEARCH

(Multiple Choice Questions for SDE Candidates)

1. An $\in$-*nbd* of $x_0 \in R^1$ is :

   (A) $\{x_0\}$.
   (B) $(x_0 - \in, x_0 + \in)$.
   (C) $(x_0 + \in, x_0 - \in)$.
   (D) $(-\in, \in)$.

2. An optimum solution to the General Linear Programming Problem is :

   (A) Any feasible solution to a General L.P.P.
   (B) Any feasible solution which optimizes the objective function.
   (C) Any solution to a General L.P.P. which satisfies the non-negative restrictions.
   (D) A particular solution to a General L.P.P. which satisfies the non-negative restrictions.

3. A feasible solution to an L.P.P. which is also a basic solution to the problem is called :

   (A) An optimum solution to the L.P.P.
   (B) A standard solution to the L.P.P.
   (C) A basic feasible solution to the L.P.P.
   (D) A feasible solution to the L.P.P.

4. The set of feasible solutions to an L.P.P. is :

   (A) An open set.
   (B) Not a convex set.
   (C) A convex set.
   (D) A cone.

5. If A is the constraint co-efficient matrix associated with primal and B is the constraint co-efficient matrix associated with dual, then :

   (A) $B = A^T$.
   (B) $B = A$.
   (C) $B = A^{-1}$.
   (D) $A = B^{-1}$.

6. A balanced transportation problem has :

   (A) An optimal solution always.
   (B) No solution.
   (C) No feasible solution.
   (D) No optimal solution.

7. If the primal is of maximization type, then the dual is :

   (A) Minimization type.

   (B) Maximization type.

   (C) Either maximization or minimization type.

   (D) None of the above.

8. A General Linear Programming Problem consists of :

   (A) An objective function.

   (B) A set of constraints.

   (C) An objective function and a set of constraints.

   (D) None of the above.

9. The optimal solution to an L.P.P. is :

   (A) Always infinite. (B) Always finite.

   (C) Unique. (D) Either unique or infinite.

10. A feasible solution to a T.P. is basic if and only if the corresponding cells in the transportation table :

   (A) Do not contain a loop. (B) Contain a loop.

   (C) Contain zero vectors. (D) None of the above.

11. If v_a is a vertex of a graph, then the set formed by v_a and all other vertices which are connected to v_a by chains, and the set of arcs connecting them, forms a ———— of the graph.

   (A) Circuit. (B) Arborescence.

   (C) Component. (D) Centre.

12. A tree with a centre is called :

   (A) An arborescence. (B) A cycle.

   (C) A circuit. (D) A chain.

13. Number of moves in a game is :

   (A) Finite. (B) Infinite.

   (C) May be finite or infinite. (D) Denumerable.

14. A matrix game is :

(A) Zero-sum. (B) Two-person.

(C) A zero-sum two person game. (D) Un-balanced.

15. Let $x \in E_n$ and let $g_i(x)$ $i = 1, 2, \ldots\ldots, m$ be convex functions in E_n. Let $S \subseteq E_n$ be the set of points satisfying the constraints $g_i(x) \leq 0, i = 1, 2, \ldots, m$. Then :

(A) S is a concave set. (B) S is a convex set.

(C) S is either concave or convex. (D) None of these.

16. A set of cells L in the transportation array is said to constitute a loop if in every row or column of the array the number of cells belonging to the set is :

(A) Zero. (B) One.

(C) Two. (D) Either zero or two.

17. In $R^3, \left\{(x_1, x_2, x_3) \in R^3, x_1^2 + x^2_2 + x^2_3 \leq 1\right\}$ is :

(A) Convex. (B) Concave.

(C) A half space. (D) Unbounded.

18. If $A \subset E_n$, then the convex hull of A is the :

(A) Largest convex set containing A.

(B) Intersection of all convex set containing A.

(C) Union of all convex set containing A.

(D) Complement of A.

19. A circular disc in a plane is :

(A) Concave set. (B) Convex set.

(C) A half space. (D) A hyperplane.

20. Spanning tree of a graph is :

(A) Unique. (B) Infinite.

(C) Not a tree. (D) Not unique.

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Mathematics

MTH 2C 10—OPERATIONS RESEARCH

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

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Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Prove that $f(x) = x_1^2 + x_2^2 + 4x_3^2 - 2x_1x_2$ is a convex function.
2. If f(X) is minimum at more than one of the vertices of S_F, then show that it is minimum at all those points which are the convex linear combinations of these vertices.
3. Write the following LP in the standard form :

Minimize $f = x_1 + x_2 - x_3$

subject to $2x_1 + 3x_2 + x_3 \geq 1$

$x_1 + 2x_2 - x_3 \leq 2$

$3x_1 + 2x_2 - x_3 = 5$

$x_1 \geq 0, x_2 \geq 0, x_3$ is unrestricted.

4. Show that the dual of the dual is the primal.

5. State the complementary slackness conditions.

6. Find the optimal strategies and the value of the game $\begin{pmatrix} -4 & 6 & 3 \\ -3 & -3 & 4 \\ 2 & -3 & 4 \end{pmatrix}$.

7. Solve the game $\begin{pmatrix} 1 & 7 \\ 6 & 2 \end{pmatrix}$.

8. Briefly explain the deletion of a variable while determining a new optimal solution from the optimal solution already obtained.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each of the following* 3 *units.*
Each question carries 2 *weightage.*

Unit I

9. Using graphical method solve the following linear programming problem.

Maximize $f(x) = 3x_1 + 5x_2$

subject to
$$x_1 + 2x_2 \le 20$$
$$x_1 + x_2 \le 15$$
$$x_2 \le 6$$
$$x_1 \ge 0, x_2 \ge 0.$$

10. Show that a vertex of the set S_F of feasible solutions is a basic feasible solution.

11. Characterize all convex functions f (X) which are defined in a convex domain $K \subseteq E_n$ and are differentiable.

Unit II

12. Explain by an example the method of finding a basic feasible solution for a transportation problem.

13. Show that the transportation problem has a triangular basis.

14. Explain the dual simplex method using the following example :

Minimize $f = 3x_1 + 5x_2 + 2x_3$

subject to $-x_1 + 2x_2 + 2x_3 \geq 3$

$x_1 + 2x_2 + x_3 \geq 2$

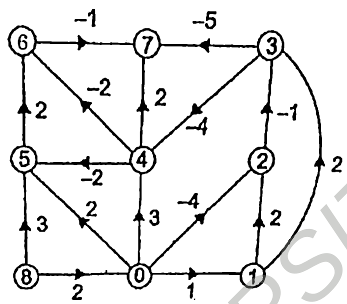
$-2x_1 - x_2 + 2x_3 \geq -4$

$x_1, x_2, x_3 \geq 0.$

Unit III

15. State and prove the fundamental theorem of rectangular games.

16. Find the minimum path from V_0 to V_7 in the following graph :



17. Minimize $f = 3x_4 + 4x_5 + 5x_6$

subject to $2x_1 - 2x_4 - 4x_5 + 2x_6 = 3$

$2x_2 + 4x_4 + 2x_5 - 2x_6 = 5$

$x_3 - x_4 + x_5 + x_6 = 4$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

x_1, x_2 integers.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. Explain the simplex method by solving the following linear programming problem :

Maximize $f(x) = 4x_1 + 5x_2$

subject to $x_1 - 2x_2 \leq 2$

$2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 5$

$-x_1 + x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0.$

19. Solve the following transportation problem :

	D_1	D_2	D_3	
O_1	2	1	3	10
O_2	4	5	7	25
O_3	6	0	9	25
O_4	1	3	5	30
	20	20	15	

20. Write an algorithm to find a minimum spanning tree and prove it.

21. Explain the cutting plane method with an example.

(2 × 5 = 10 weightage)

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

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MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(Multiple Choice Questions for SDE Candidates)

1. The general solution of the differential equation $4y^{11} + 4y^{1} - 3y = 0$ is :

   (A) $y = Ae^{x/2} + Be^{-3x/2}$. (B) $y = Ae^{x} + Be^{-x}$.

   (C) $y = Ae^{x/4} + Be^{-x}$. (D) $y = A\sin x + B\cos x$.

2. The general solution of the differential equation $x^2y^{11} - 4xy^{1} + 6y = 0$ is :

   (A) $y = Ax^2 + Bx^3$. (B) $y = Ae^{2x} + Be^{3x}$.

   (C) $y = A\sin 3x + B\cos 2x$. (D) $y = A\cos 3x + B\sin 2x$.

3. Solution of the differential equation $y^1 = y$ is :

   (A) $y = \sin x$. (B) $y = \cos x$.

   (C) $y = \tan^{-1} x$. (D) $y = e^x$.

4. Origin is a ——————— point of the differential equation $\left(1 - x^2\right)y^{11} - 2xy^{1} + 2y = 0$.

   (A) Ordinary. (B) Singular.

   (C) Regular singular. (D) Limit.

5. The differential equation $y^{11} + 7y^{1} - 8y = 0$ has :

   (A) Only one independent solution. (B) Two independent solutions.

   (C) Three independent solutions. (D) Infinite number of independent solutions.

6. The general solution of the differential equation $y^1 = \cos x$ is :

   (A) $y = \sin x$. (B) $y = \cos x$.

   (C) $y = c\sin x$. (D) $y = \sin x + c$.

7. The singular points of the Gauss's hyper geometric equation are :

   (A) $x = 0$ and $x = 1$. (B) $x = 0$ and $x = -1$.

   (C) $x = 0$ and $x = 2$. (D) $x = 1$ and $x = -1$.

8. $\lim_{b \to \infty} F\left(a, b, a, \frac{x}{b}\right)$ equals :

(A) $(1+x)^p$. (B) $\sin^{-1} x$.

(C) $\text{Log}\,(1+x)$. (D) e^x.

9. Using Rodrigue's formula, the n^{th} Legendre polynomial $P_n(x)$ is exposed as :

(A) $P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n}\left(x^2-1\right)^n$. (B) $P_n(x) = \frac{1}{n^2\, n!} \frac{d^n}{dx^n}\left(x^2-1\right)^n$.

(C) $P_n(x) = \frac{d^n}{dx^n}\left(x^2-1\right)^n$. (D) $P_n(x) = \frac{1}{n!}\left(x^2-1\right)^n$.

10. $P_2(x)$, the Legendre polynomial of order '2' is :

(A) x. (B) $\frac{1}{2}\left(3x^2-1\right)$.

(C) 1. (D) $3x^2$.

11. The Bessel function of the first kind of order 'P' is given by $J_p(x) =$

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n \left(x/2\right)^{2n+p}}{n!\,(p+n)!}$. (B) $\sum_{n=0}^{\infty} \frac{\left(x/2\right)^{2n+p}}{n!\,(n+p)!}$.

(C) $\sum_{n=0}^{\infty} \frac{(-1)^{n+p} \left(x/2\right)^{n+p}}{n!\,(p+n)!}$. (D) $\sum_{n=0}^{\infty} \frac{(-1)^n \left(x/2\right)^{2p+n}}{n!\,(n+p)!}$.

12. The general solution of the system $\frac{dx}{dt} = x + y, \frac{dy}{dt} = 4x - 2y$ is :

(A) $x = c_1e^{-3t} + c_2e^{2t}, y = -4c_1e^{-3t} + c_2e^{2t}$. (B) $x = c_1e^{-t} + c_2e^{2t}, y = 4c_1e^{-t} + c_2e^{2t}$.

(C) $x = c_1e^{3t} + c_2e^{-2t}, y = 4c_1e^{3t} + c_2e^{-2t}$. (D) $x = c_1e^{t} + c_2e^{2t}, y = 4c_1e^{t} + c_2e^{2t}$.

13. The only critical point of the system $\frac{dx}{dt} = -y, \frac{dy}{dt} = x$ is :

(A) $x = 1, y = 1$. (B) $x = 2, y = 2$.

(C) $x = -1, y = -1$. (D) $x = 0, y = 0$.

14. In a physical system, if the total energy has a local minimum at certain equilibrium point, then it is :

(A) Unstable.

(B) Always origin.

(C) Total energy always zero at that point.

(D) Stable.

Turn over

15. If $u(x)$ is a non-trivial solution of $u^{11} + q(x)u = 0, q(x) > 0$ for all $x > 0$ and if $\int_1^{\infty} q(x)\,dx = \infty$, then $u(x)$ has ———————— zeros on the positive x-axis.

(A) Exactly one. (B) No.

(C) Exactly two. (D) Infinitely may.

16. If $y(x)$ and $z(x)$ are non-trivial solution of $y^{11} + q(x)y = 0$ and $z^{11} + y(x)z = 0$ where $q(x)$ and $r(x)$ are positive functions such that $q(x) > r(x)$. Then $y(x)$ vanishes :

(A) Exactly twice between any two successive zeros of $z(x)$.

(B) Exactly once between any two successive zeros of $z(x)$.

(C) Atleast once between any two successive zeros of $z(x)$.

(D) Atmost once between any two successive zeros of $z(x)$.

17. First approximation to the solution of the problem $y^1 = y, y(0) = 1$, by Picard's method is :

(A) $y_1(x) = 1 + x$. (B) $y_1(x) = 1 + x^2$.

(C) $y_1(x) = 1 + \frac{x^2}{2!}$. (D) $y_1 = 1$.

18. First approximation to the solution of the initial value problem $y^1 = 2x(1+y)$ with $y(0) = 0$ is :

(A) $y_1 = 1$. (B) $y_1 = 1 + x$.

(C) $y_1 = x^2$. (D) $y_1 = 1 + x^2$.

19. The closed plane curve of given length that endoses largest area is :

(A) Rectangle. (B) Parallelogram.

(C) Circle. (D) Triangle.

20. Euler's differential equation for an extremal is :

(A) $\frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) - \frac{\partial f}{\partial y} = 0$. (B) $\frac{d}{dx}\left(\frac{\partial^2 f}{\partial x\,\partial y^1}\right) - \frac{\partial f}{\partial y} = 0$.

(C) $\frac{d}{dx}\left(\frac{\partial f}{\partial x}\right) - \frac{\partial f}{\partial y} = 0$. (D) $\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial^2 f}{\partial y^2} = 0$.

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Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Find the value of a_0 in the Fourier series expansion of $f(x) = x, -\pi \le x \le \pi$.
2. Write the Legendre's differential Equation.
3. State Dirichlet's theorem.
4. Find co-efficient a_n in the cosine series expansion for the function $f(x) = \sin x,\ 0 \le x \le \pi$.
5. Find the general solution of $y'' + 9y = 0$.

Turn over

6. Find the Fourier series expansion of the function $f(x) = \cos^2 x$.

7. Write the Rodrigue's formula for the Legendre polynomials.

8. Define Lipschitz Continuity of a function. Give an example.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. Solve $xy'' - (x+2)y' + 2y = 0$.

10. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n$.

11. Show that $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$.

Unit II

12. Show that $\Gamma(p+1) = p\Gamma(p)$.

13. Show that the functions e^{3t} and e^{2t} are linearly independent.

14. Find the second Picard approximation for the solution of the equation $\frac{du}{dx} = u^2 + x$ given that $u = 0$ when $x = 0$.

Unit III

15. Test for linear independence of the functions $\sin x, \sin 3x, \sin^3 x$.

16. Express $x^4 - 3x^2 + x$ in terms of Legendre polynomials.

17. Prove that $P_n(x) = 2F_1\left(-n, n+1, 1, \frac{1-x}{2}\right)$.

(6 × 2 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. Solve in series $\left(1-x^2\right) y'' - xy' + 4y = 0$.

19. If $m > (n-1)$ and n is a positive integer, show that

$$\int_0^1 x^m \, P_n(x)\, dx = \frac{m(m-1)(m-2)\ldots\ldots(m-n/2)}{(m+n+1)(m+n-1)\ldots.(m-n+3)}.$$

20. Show that $J_n(-x) = (-1)^n J_n(x)$.

21. Extremize the functional $V\left[y(x)\right] = \int_{x_0}^{x_1} \frac{1+y^2}{(y')^2}\, dx$.

(2 × 5 = 10 weightage)

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

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MTH 2C 08—TOPOLOGY

(Multiple Choice Questions for SDE Candidates)

1. Let X be a set with cofinite topology and x_n be a sequence in X. Then x_n is convergent in X if and only if :

   (A) There is atmost one term in x_n that repeats infinitely.

   (B) x_n only has finitely many distinct terms.

   (C) x_n is eventually constant.

2. If U is open in X and A is closed in X, then :

   (A) $A \setminus U$ is open in X. (B) $A \setminus U$ is closed in X.

   (C) $A \setminus U$ is both open and closed in X.

3. In the cofinite topology on R, to what point or points does the sequence $x_n = 1/n$ converge :

   (A) 0. (B) 1.

   (C) Almost every point in R.

4. The closure of $A \cup B$ is :

   (A) $\bar{A} \cup \bar{B}$. (B) $\bar{A} \cap \bar{B}$.

   (C) $(A \cap B) \cup (A \cap \bar{B})$.

5. The interior of an empty set φ is :

   (A) φ. (B) Not defined.

   (C) The entire space.

6. Consider the R with usual topology. What are the accumulation points of set of rational numbers ?

   (A) Set of all integers. (B) Set of irrational numbers.

   (C) R.

7. Which of the following statements about continuous functions are true ?

   (A) Inverse of continuous function is always continuous.

   (B) Continuous functions is always one-one.

   (C) Composition of continuous functions are continuous.

8. When is a topological space X is said to be compact ?

   (A) There exits a finite open cover for X.

   (B) Every open cover of X has a finite subcover.

   (C) If and only if X has finite many elements.

9. A space X is said to be connected if there exists no non-empty subsets A and B of it such that :

   (A) $A \cup B = X, A \cap B = \varphi$ and A, B are both open in X.

   (B) $A \cup B = X, A \cap B = \varphi$.

   (C) $A \cup B = X$ and A, B are both open in X.

10. The product of two path connected spaces necessarily connected :

   (A) True. (B) False.

   (C) Cant say.

11. Which of the statements below is false ?

   (A) The real line with the usual topology is locally connected.

   (B) Topologist's sine curve is locally connected but not connected.

   (C) Every quotient space of a locally connected space is locally connected.

12. A metric space (X, d) with the associated metric topology, where X contains at least two elements is Hausdorff :

   (A) True. (B) False.

   (C) Need not be.

13. Which the following statements are true ?

   (A) In a T_1 space, limits of sequences are unique.

   (B) Every T_2 space is metrisable.

   (C) The real line with the semi-open interval topology is T_2.

14. Which of the following statement is true ?

   (A) there exists spaces which satisfies T_4-axion but fails to satisfy T_3-axiom.

   (B) Metric spaces need not be T_3.

   (C) Every Tychonoff space is T_3.

15. Every metric space can also be seen as a topological space :

(A) True. (B) False.

16. R with the usual topology is a connected topological space :

(A) True. (B) False.

17. Finite topological spaces are always connected :

(A) True. (B) False.

18. The topology determined by the basis $\beta = \{(a, b) : a < b\}$:

(A) K. (B) K 0{ }.

(C) R.

19. Is R with usual topology is a compact topological space :

(A) True. (B) False.

20. If finitely many points from the set $D = \{(x, y) : x^2 + y^2 \leq 1\}$, is the resulting set compact.

(A) True. (B) False.

(C) Cant say.

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Part A

Answer **all** *questions.*
Each question carries weightage 1.

1 Define neighbourhood of a point in a topological space. Prove that a subset of a topological space is open iff it is a neighbourhood of each of its points.

2 Explain Sierpinski space. Prove that the Sierpinski space is not obtainable from a pseudometric.

3 Let $(X, \mathfrak{T})$ be a topological space and let $A \subset X$. Then prove that $\overline{\overline{A}} = \overline{A}$.

4 Prove that the property of being a discrete space is divisible.

5 Prove that compactness is preserved under continuous functions.

6 In a Hausdorff space, prove that limits of sequences are unique.

Turn over

7. State Urysohn's lemma.

8. Let X be a topological space and (Y, d) be a metric space. Let $\{f_n\}$ be a sequence of functions from X to Y converging uniformly to $f : X \to Y$. Then prove that if each f_n is continuous, so is f.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit.*
Each question carries weightage 2.

Unit I

9. Let $(X, \mathfrak{T})$ be a topological space and S be a family of subsets of X. Then prove that S is a sub-base of X iff S generates $\mathfrak{T}$.

10. For a subset A of a space X, prove that :

$\bar{A} = \{y \in X : \text{every neighbourhood of } y \text{ meets A non-vacuously}\}$.

11. Let $(X, \mathfrak{T})$ and $(Y, \mathfrak{U})$ be topological spaces and let $f : X \to Y$ be any function. If f is continuous at $x_0 \in X$, then prove that for every subset A of X, $x_0 \in \bar{A}$ implies $f(x_0) \in \overline{f(A)}$.

Unit II

12. Let $\mathcal{C}$ be a collection of connected subsets of a space X such that no two members of $\mathcal{C}$ are mutually separated. Then prove that union of all elements of $\mathcal{C}$ is connected.

13. Prove that the product topology is the weak topology determined by the projection functions.

14. Prove that every quotient space of a locally connected space is locally connected.

Unit III

15. Prove that a space X is a T_1 space iff every finite subset of X is closed.

16. Prove that all metric spaces are T_4.

17. Prove that there can be no continuous one-to-one map from the unit circle S^1 into the real line.

(6 × 2 = 12 weightage)

Part C

*Answer any **two** questions.*
Each question carries weightage 5.

18. (a) For a subset A of a topological space X, prove that $\bar{A} = A \cup A'$.

(b) Let $\{(X_i, \mathfrak{T}_i) : i = 1, 2, \ldots n\}$ be a collection of topological spaces and $(X, \mathfrak{T})$ their topological product. Then prove that each projection π_i is continuous. Moreover, if Z is any space, then prove that a function $f : Z \to X$ is continuous iff $\pi_i \circ f : Z \to X_i$ is continuous for all $i = 1, 2, \ldots n$.

19. (a) Let (X, d) be a compact metric space and let $\mathfrak{U}$ be an open cover of X. Then prove that there exist a positive real number r such that for any $x \in X$ there exist $V \in \mathfrak{U}$ such that $B(x, r) \subset V$.

(b) Prove that every second countable space is first countable. Is the converse true ? Justify your answer.

20. (a) In a topological space X, prove the following :

(i) Components are closed sets.

(ii) Any two distinct components are mutually disjoint.

(iii) Every non-empty connected subset is contained in a unique component.

(iv) Every space is the disjoint union of its components.

(b) Prove that every open subset of the real line in the usual topology can be expressed as the union of mutually disjoint open intervals.

21 State and prove the Tietze extension theorem.

($2 \times 5 = 10$ weightage)

C 23354-A (Pages : 4) Name..

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 07—REAL ANALYSIS—II

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 2C 07—REAL ANALYSIS—II

(Multiple Choice Questions for SDE Candidates)

1. If X is a set and A is an algebra of subsets of X, then which one of the following is true ?

   (A) $P(X) \subseteq A \subseteq \{\emptyset, X\}$.
   (B) $A \subset P(X) \subset \{\emptyset, X\}$.
   (C) $A \subset \{\emptyset, X\} \subset P(X)$.
   (D) $\{\emptyset, X\} \subseteq A \subseteq P(X)$.

2. Which one of the following statements is false :

   (A) G_δ sets are Borel set.
   (B) A G_δ set need not be a Borel set
   (C) F_σ sets are Borel set.
   (D) A countable intersection of open sets is a Borel set.

3. Which one of the following statements is true.

   (A) Outer mesure of $\left(\bigcup_{k=1}^{\infty} E_k\right) \le \sum_{k=1}^{\infty}$ (outer mesure of E_k).

   (B) Outer mesure of $\left(\bigcup_{k=1}^{\infty} E_k\right) \ge \sum_{k=1}^{\infty}$ (outer mesure of E_k).

   (C) Outer mesure of $\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty}$ (outer mesure of E_k).

   (D) Outer mesure of $\left(\bigcup_{k=1}^{\infty} E_k\right) > \sum_{k=1}^{\infty}$ (outer mesure of E_k).

4. Let A be any set, and $\{E_1, ..., E_n\}$ be a disjoint collection of measurable sets. Then ————.

   (A) $m^*\left(A \cap \left[\bigcup_{k=1}^{n} E_k\right]\right) \ge \sum_{k=1}^{n} m^*(A \cap E_k)$.
   (B) $m^*\left(A \cap \left[\bigcup_{k=1}^{n} E_k\right]\right) > \sum_{k=1}^{n} m^*(A \cap E_k)$.
   (C) $m^*\left(A \cap \left[\bigcup_{k=1}^{n} E_k\right]\right) < \sum_{k=1}^{n} m^*(A \cap E_k)$.
   (D) $m^*\left(A \cap \left[\bigcup_{k=1}^{n} E_k\right]\right) = \sum_{k=1}^{n} m^*(A \cap E_k)$.

5. Which one of the following is not true ?

   (A) The translate of a measurable set is measurable.
   (B) The translate of a measurable set need not be measurable.
   (C) The Borel σ-algebra is contained in every σ-algebra that contains all open sets.
   (D) The Borel σ-algebra is the intersection of all the σ-algebras of sub-sets of $\mathbb{R}$ that contain the open sets.

6. A countable collection of sets $\{E_k\}_{k=1}^{\infty}$ is said to be descending provided ————.

   (A) For each k, $Ek = E_{k+1}$.
   (B) For each k, $E_k \subseteq E_{k+1}$.
   (C) For each k, $E_k \supseteq E_{k+1}$.
   (D) None of the above options.

7. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of measurable sets for which $\sum_{k=1}^{\infty} m(E_k) < \infty$. Then ——————.

   (A) Almost all $x \in \mathbb{R}$ belong to all E_k's.

   (B) All $x \in \mathbb{R}$ belong to at most finitely many of the E_k's.

   (C) Almost all $x \in \mathbb{R}$ belong to at most finitely many of the E_k's.

   (D) None of the above options.

8. Which one of the following is true ?

   (A) The Cantor set C is an open, uncountable set of measure zero.

   (B) The Cantor set C is a closed, uncountable set of measure zero.

   (C) The Cantor set C is an open, countable set of measure zero.

   (D) The Cantor set C is a closed, countable set of measure zero.

9. Which one of the following is false ?

   (A) Let f and g be measurable functions on E that are finite a.e. on E. For any α and β, $\alpha f + \beta g$ is measurable on E.

   (B) Let f and g be measurable functions on E that are finite a.e. on E. Then $f + g$ is measurable on E.

   (C) Let f and g be measurable functions on E that are finite a.e. on E. Then fg is measurable on E.

   (D) Let f and g be measurable functions on E that are finite a.e. on E. Then fg need not be measurable on E.

10. The characteristic function χ_A is measurable if and only if ——————.

   (A) A has measure 0.

   (B) A has measure ∞.

   (C) A is measurable.

   (D) None of the above options.

11. The upper Darboux sum for f with respect to the partition $P = \{x_0, x_1, \ldots, x_n\}$ of $[a, b]$, denoted by $U(f, P)$, is given by ——————.

   (A) $U(f, P) = \sum_{i=1}^{n} M_{i-1} x_i$, where, for $1 \le i \le n$, $M_i = \sup\{f(x) \mid x_{i-1} < x < x_i\}$.

   (B) $U(f, P) = \sum_{i=1}^{n} M_i x_i$, where, for $1 \le i \le n$, $M_i = \sup\{f(x) \mid x_{i-1} < x < x_i\}$.

   (C) $U(f, P) = \sum_{i=1}^{n} M_i \cdot (x_i - x_{i-1})$ where, for $1 \le i \le n$, $M_i = \sup\{f(x) \mid x_{i-1} < x < x_i\}$.

   (D) $U(f, P) = \sum_{i=1}^{n} M_{i-1} x_{i-1}$, where, for $1 \le i \le n$, $M_i = \sup\{f(x) \mid x_{i-1} < x < x_i\}$.

12. A bounded real-valued function f defined on the closed, bounded interval $[a, b]$ is Riemann integrable over $[a, b]$ if ——————.

   (A) $(R)\underline{\int}_a^b f < (R)\overline{\int}_a^b f$.

   (B) $(R)\underline{\int}_a^b f = (R)\overline{\int}_a^b f$.

   (C) $(R)\underline{\int}_a^b f > (R)\overline{\int}_a^b f$.

   (D) $(R)\underline{\int}_a^b f \ge (R)\overline{\int}_a^b f$.

13. Let f be a bounded measurable function on a set of finite measure E. Suppose A and B are disjoint measurable subsets of E. Then $\int_{A \cup B} f =$ ————.

(A) $\int_A f - \int_B f$. (B) $\int_B f - \int_A f$.

(C) $\int_A f + \int_B f$. (D) None of the above options.

14. Let f be a non-negative measurable function on E. Then $\int_E f = 0$ if and only if ————.

(A) $f \leq 0$ *a.e.* on E. (B) $f = 0$ *a.e* on E.

(C) $f \geq 0$ *a.e* on E. (D) $f > 0$ *a.e* on E.

15. Let the non-negative function f be integrable over E. Then ————.

(A) f is finite a.e. on E. (B) f is 0 a.e. on E.

(C) f is constant a.e. on E. (D) None of the above options.

16. Let the functions f and g be integrable over E. Then for any α and β, ————.

(A) $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$. (B) $\int_E (\alpha f + \beta g) = \alpha \int_E f - \beta \int_E g$.

(C) $\int_E (\alpha f + \beta g) = -\alpha \int_E f + \beta \int_E g$. (D) $\int_E (\alpha f + \beta g) = -\alpha \int_E f - \beta \int_E g$.

17. A family F of measurable functions on E is said to be uniformly integrable over E provided ————.

(A) For each $\varepsilon > 0$, there is a $\delta > 0$ and an $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \varepsilon$.

(B) For each $\varepsilon > 0$, there is a $\delta > 0$ such that for each $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \varepsilon$.

(C) There is a $\varepsilon > 0$, there is a $\delta > 0$ such that for each $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \varepsilon$.

(D) None of the above options.

18. Let f be a monotone function on the open interval (a, b). Then f is :

(A) Continuous at all points in (a, b).

(B) Continuous at all rational points in (a, b).

(C) Continuous at all irrational points in (a, b).

(D) Continuous except possibly at a countable number of points in (a, b).

19. If $\overline{D} f(x)$ denotes the upper derivative of f at x, and $\underline{D} f(x)$ denotes the lower derivative of f at x, then always ————.

(A) $\overline{D} f(x) \geq \underline{D} f(x)$. (B) $\overline{D} f(x) = \underline{D} f(x)$.

(C) $\overline{D} f(x) \leq \underline{D} f(x)$. (D) $\overline{D} f(x) < \underline{D} f(x)$.

20. If the function f is monotone on the open interval (a, b), then it is ————.

(A) differentiable at rational points in (a, b). (B) differentiable on (a, b).

(C) differentiable almost everywhere on (a, b). (D) differentiable at irrational points in (a, b).

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 07—REAL ANALYSIS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

Covid Instructions are not applicable for Pvt/SDE students

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Define Lebesgue outer measure and show that it is monotone.
2. Prove that translate of a measureable set is measureable.
3. State and prove a necessary condition for a function f to be measureable.
4. Define Riemann integrability and show that the Dirichlet's function is not Riemann integrable.
5. Show by an example that strict inequality can occur in Fatou's Lemma.
6. When can we say that a family of measurable functions on E to be is tight over E. State the Vitali Convergence Theorem for sequence of functions defined on any set E.
7. Define functions of bounded variation and prove that monotone functions are of bounded variation.
8. State and Prove Riesz - Fischer Theorem.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **six** *question, choosing two questions from each unit.*
Each question has weightage 2.

UNIT I

9. Prove that Lebesgue outer measure is countably sub additive.

10. State and Prove the Outer approximation of a measureable set by Open sets and G_δ sets.

11. State and Prove Simple Approximation Lemma.

UNIT II

12. State and prove Monotone convergence Theorem.

13. Define Uniform integrability of a family of measureable functions on E and show that point wise limit of a sequence of uniformly integrable functions on E is integrable.

14. Prove that a bounded function defined on a set of finite measure E is Lebesgue integrable over E if and only if it is measurable.

UNIT III

15. Prove that a monotone function on (a, b) is continuous except at a countable number of points in its domain.

16. Prove that a function is of bounded variation on a closed bounded interval $[a, b]$ if and only if it is the difference of two increasing functions on $[a, b]$.

17. State and prove Holder inequality and derive Cauchy Schwarz Inequality.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *from the following four questions (18-21).*
Each question has weightage 5.

18. Define a Sigma Algebra and prove the class of all Lebesgue measureable sets forms sigma algebra.

19. Define measurability of functions and prove that the sum and product of two measurable and finite valued functions on E are measurable.

20. Let f and g are non negative functions defined on E. Then

    a) State and prove linearity property of integration.

    b) State and prove Monotonicity property of integration.

    c) State and prove Additivity over domains of integration.

21. State and prove Vitali Covering Lemma.

(2 × 5 = 10 weightage)

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 06—ALGEBRA—II

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 2C 06—ALGEBRA—II

(Multiple Choice Questions for SDE Candidates)

1. Identity element of the group R under addition is :

(A) 1. (B) 2.

(C) 0. (D) –1.

2. Every group of order 4 is :

(A) Cyclic. (B) Abelian.

(C) Non-abelian. (D) None of the Above.

3. Characteristic of Ring R is :

(A) 3. (B) 2.

(C) 1. (D) 0.

4. Find characteristic of $Z_3 \times 3Z$:

(A) 4. (B) 2.

(C) 3. (D) 0.

5. Which of the following is a Maximal ideal of Z ?

(A) 4Z. (B) Z_2.

(C) Q. (D) 2Z.

6. Which of the following is not a field ?

(A) Z. (B) R.

(C) Q. (D) C.

7. Which of the following is an example of Transcendental number ?

(A) $\sqrt{2}$. (B) π.

(C) i. (D) 2.

8. Number of ideals of a Field is :

(A) 0. (B) 2.

(C) 1. (D) 3.

9. Which of these are not constructible numbers ?

(A) $\sqrt{2}$.

(B) π.

(C) $\sqrt{3}$.

(D) 4.

10. Which of the following is not an order of a finite field ?

(A) 16.

(B) 20.

(C) 3.

(D) 5.

11. Find dimension of R over Q is :

(A) 1.

(B) 3.

(C) ∞.

(D) 2.

12. The field $Q\left(\sqrt{3}+\sqrt{7}\right)$ is isomorphic to :

(A) Q.

(B) R.

(C) $Q\left(\sqrt{3}, \sqrt{7}\right)$.

(D) C.

13. Find the fixed field of $Q\left(\sqrt{2}\right)$ of the mapping $\sqrt{2}$ goes to $-\sqrt{2}$:

(A) Q.

(B) R.

(C) C.

(D) Z.

14. Find Galois group of the polynomial $x^4 - 5x^2 + 6$ over Q :

(A) Z_2.

(B) Z_3.

(C) Klein 4 group.

(D) Z_5.

15. Which of the following is not a Fermat prime :

(A) 3.

(B) 5.

(C) 17.

(D) 8.

16. Find the number of Quadratic polynomials which is irreducible over Z_2 :

(A) 3. (B) 1.

(C) 4. (D) 8.

17. Which of the following is an Abelian group :

(A) A_3. (B) A_4.

(C) S_3. (D) D_4.

18. Find number of elements of order 2 in S_3 :

(A) 2. (B) 6.

(C) 3. (D) 5.

19. Find number of proper subgroups of S_3 :

(A) 5. (B) 4.

(C) 3. (D) 1.

20. The number of normal sub-groups in a non-trivial simple group :

(A) 2. (B) 1.

(C) 0. (D) 4.

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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 06—ALGEBRA—II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

Covid Instructions are not applicable for SDE/Private Students

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Let R be a finite commutative ring with unity. Show that every prime ideal in R is a maximal ideal.
2. Show that $\sqrt{1+\sqrt[3]{2}}$ is algebraic over $\mathbb{Q}$.
3. Find a basis for $Q\left(\sqrt{2},\sqrt{3}\right)$ over $\mathbb{Q}$.
4. Find the number of primitive 8th roots of unity in GF (9).
5. What is the order of $G\left(Q\left(\sqrt[3]{2},i\sqrt{3}\right)/Q\left(\sqrt[3]{2}\right)\right)$?
6. Let E be a finite extension of the field F. Show that if E is separable over F then each α in E is separable over F.

Turn over

7. Show that the regular 7– gon is not constructible.

8. Find the 6th cyclotomic polynomial over the rational Q.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each of the following* 3 *units.*
Each question carries 2 *weightage.*

Unit I

9. Prove that $Q\left(2^{1/2}, 2^{1/3}\right) = Q\left(2^{1/6}\right)$.

10. Prove that a field F is algebraically closed if and only if every non-constant polynomial in F [x] factors in F [x] into linear factors.

11. Show that trisecting the angle is impossible.

Unit II

12. Let E be a field of p^n elements contained in an algebraic closure $\overline{\mathbb{Z}}_p$ of $\mathbb{Z}_p$. Show that the elements of E are precisely the zeros in $\mathbb{Z}_p$ of the polynomial $x^{p^n} - x$ in $\mathbb{Z}_p[x]$.

13. Let $\overline{F}$ be algebraic closure of F and $E \leq \overline{F}$ be a splitting field over F. Prove that every isomorphism of E into $\overline{F}$ leaving elements of F fixed is an automorphism of E.

14. Prove that If E is a finite extension of F, then {E : F} divides [E : F].

Unit III

15. State the Main Theorem of Galois Theory.

16. Show that Galois group of the n^{th} cyclotomic extension of Q has $\phi(n)$ elements and is isomorphic to the group consisting of the positive integers less than n and relatively prime to n under multiplication modulo n.

17. Let $y_1, y_2, \ldots, y_5$ be independent transcendental real numbers over Q. Show that the polynomial $f(x) = \prod_{i=1}^{5}(x - y_i)$ is not solvable by radicals over $F = Q(s_1, s_2, \ldots, s_5)$, where s_i is the i^{th} elementary symmetric function in $y_1, y_2, \ldots, y_5$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. (a) Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is a field.

(b) Find a non-trivial proper ideal of $\mathbb{Z}\times\mathbb{Z}$ that is not prime.

19. (a) Let E be an algebraic extension of a field F. Show that there exist a finite number of elements $\alpha_1,\alpha_2,\ldots,\alpha_n$ in E such that $E=F(\alpha_1,\alpha_2,\ldots,\alpha_n)$ if and only if E is a finite extension of F.

(b) Let E be an extension of a field F. Show that the set G (E/F) of all automorphisms of E leaving F fixed is a group under function composition and the fixed field of G (E/F) contains F.

20. (a) Let F be a field, and let α and β be algebraic over F with $\deg(\alpha, F)=n$. Prove that the map

$\psi_{\alpha,\beta}:F(\alpha)\to\beta$ defined by $\psi_{\alpha,\beta}\left(c_0+c_1\alpha+\ldots+c_{n-1}\alpha^{n-1}\right)=c_0+c_1\beta+\ldots+c_{n-1}\beta^{n-1}$ for $c_i\in F$ is

an isomorphism of $F(\alpha)$ onto $F(\beta)$ if and only if α and β are conjugate over F.

(b) Find the splitting field of the polynomial x^4+x^2+1 in $Q[x]$.

21. Let F be a field of characteristic 0, and let $a\in F$. Prove that if K is the splitting field of x^n-a over F, then G (K/F) is a solvable group.

(2 × 5 = 10 weightage)

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SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 2C 10—OPERATIONS RESEARCH

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Let $f : X \to R$ be a function, where $X \subseteq R, 1$, and let f be differentiable on X. Then f is convex if and only if ——————.
2. Define feasible solution of LPP.
3. Define a linear function.
4. How many columns in the simplex table, if there are '*m*' original variables and '*n*' introduced variables in LPP ?
5. This method of formal calculations often termed as Linear Programming was developed later in which year ?
6. What is dual of the dual in LPP ?
7. The solution of any transportation problem is obtained in how many stages.
8. What are the three methods to find the basic feasible solution of transportation problem ?
9. A transportation problem has a feasible solution, when ?
10. What does indicate the degeneracy in the transportation problem ?
11. What is pure integer programming ?
12. What is the full form of PERT and CPM ?
13. What are different types of floats ?
14. Define optimistic and pessimistic times.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Prove that a vertex of the set of feasible solutions is a basic feasible solution.

16. Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Then prove that $f(X)$ is a convex function iff $f(X_2) - f(X_1) \geq (X_2 - X_1)\nabla f(X_1)$ for all X_1, X_2 in K.

17. Solve the following LPP by graphically :

    Maximize $z = 120x + 100y$

    subject to the conditions
    $$2x + y \leq 16$$
    $$x + y \leq 11$$
    $$x + 2y \geq 6$$
    $$5x + 6y \leq 90 \text{ and } x_1, x_2 \geq 0.$$

18. Prove that the optimum value of $f(X)$ of the primal, if it exists, is equal to the optimum value of $\Phi(Y)$ of the dual.

19. What is an unbalanced transportation problem ? How do you overcome the difficulty ?

20. Determine basic feasible solution to the following transportation problem using North West Corner rule :

	1	2	3	4	5	Supply
A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	

21. A building activities has been analyzed as follows. v_j stands for a job.

    (i) v_1 and v_2 can start simultaneously, each one taking 10 days to finish.

    (ii) v_3 can start after 5 days and v_4 after 4 days of starting v_1.

    (iii) v_4 can start after 3 days of work on v_3 and 6 days of work on v_2.

    (iv) v_5 can start after v_1 is finished and v_2 is half done.

    (v) v_3, v_4 and v_5 take respectively 6, 8 and 12 days to finish.

    Find the critical path and the minimum time for completion.

22. State and prove the necessary and sufficient condition for the existence of a saddle point in game theory.

23. Prove that for an $m \times n$ matrix game both $\max_X \min_Y E(X,Y)$ and $\min_X \max_Y E(X,Y)$ exist and are equal.

24. Solve the given payoff matrix by graphical method and state optimal strategies of players A and B :

		B: I	II	III	IV	V
A	I	– 5	5	0	–1	8
	II	8	–4	–1	6	–5

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Solve the following LPP :

    Maximize $Z = 2x_1 + x_2 + \frac{1}{4}x_3$

    subject to the conditions
    $$4x_1 + 6x_2 + 3x_3 \le 8$$
    $$3x_1 - 6x_2 - 4x_3 \le 1$$
    $$2x_1 + 3x_2 - 5x_3 \ge 4$$
    $$x_1, x_2, x_3 \ge 0.$$

26. Solve the following LPP by dual simplex method :

Maximize $Z = 2x_1 + 2x_2 + 4x_3$

subject to the conditions $2x_1 + 3x_2 + 5x_3 \geq 2$

$$3x_1 + x_2 + 7x_3 \leq 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0.$$

27. Find the optimum solution to the following transportation problem :

	1	2	3	4	5	Available
A	4	3	1	2	6	80
B	5	2	3	4	5	60
C	3	5	6	3	2	40
D	2	4	4	5	3	20
Required	60	60	30	40	10	

28. Solve the following game by graphically :

B

		y_1	y_2	y_3	y_4
	X_1	19	6	7	5
A	X_2	7	3	14	6
	X_3	12	8	18	4
	X_4	8	7	13	–1

(2 × 4 = 8 weightage)

C 23147 (Pages : 3) Name.......................................

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CUCSS)

Mathematics

MT 2C 09—ODE AND CALCULUS OF VARIATIONS

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions*
Each question carries 1 *weightage.*

1. Find a power series solution of the form $\sum a_n x^n$ of the equation $y' + y = 1$.

2. Determine the nature of the point $x = 0$ for the equation $xy'' + (\sin x)\, y = 0$.

3. Define hypergeometry function and also write the singular points.

4. Determine the nature of the point $x = \infty$ for the Legendre's equation $\left(1 - x^2\right) y'' - 2xy' + p\,(p+1)\, y = 0$.

5. Show that $\frac{d}{dx}\left[J_\alpha (x)\right] = \frac{1}{2}\left(J_{\alpha - 1}(x) - J_{\alpha + 1}(x)\right)$.

6. State the Rodrigues formula.

7. Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

8. Describe the phase portrait of the system : $\frac{dx}{dt} = -x, \frac{dy}{dt} = -y$.

9. Find the critical point of the system : $\frac{dx}{dt} = y\left(x^2 + 1\right), \frac{dy}{dt} = 2xy^2$.

Turn over

10. Determine whether the function $2x^2 - 3xy + 3y^2$ is positive definite, negative definite (or) neither.

11. State Picard iteration.

12. Find the exact solution of the initial value problem $y' = y^2, y(0) = 1$. Starting with $y_0(x) = 1$, apply Picard's method to calculate $y_1(x)$.

13. Write the equation of oscillation behaviour of non trivial solution of differential equation.

14. Find the stationary function of $\int_0^4 \left[xy' - (y')^2\right] dx$ which, is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *the questions.*
Each question carries 2 weightage.

15. The equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius series solution. Find it.

16. Find the indicial equation and its roots of the equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.

17. Find the general solution of $(1 - e^x)y'' + \frac{1}{2}y' + e^x y = 0$ near the singular point $x = 0$ by changing the independent variable to $t = e^x$.

18. Find the critical point of the system

$$\begin{cases} \dfrac{dx}{dt} = 2x - 2y + 10 \\ \dfrac{dy}{dt} = 11x - 8y + 49 \end{cases}.$$

19. Find the critical points of $\frac{d^2x}{dt^2} + \frac{dx}{dt} - (x^3 + x^2 - 2x) = 0$.

20. Determine the nature of stability properties of the critical point $(0, 0)$ for the system :

$$\frac{dx}{dt} = 4x - 3y, \frac{dy}{dt} = 8x - 6y.$$

21. Let $y(x)$ be a non-trivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a, b]$. Then $y(x)$ has at most a finite number of zeroes in this interval

22. Apply the Picard's method to the initial value problem $y' = x + y, y(0) = 1$ with $y_0(x) = 1 + x$.

23. Apply the Picard's method to the initial value problem $y' = x + y, y(0) = 1$ with $y_0(x) = e^x$.

24. Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition any rectangle $a \le x \le b$ and $c \le y \le d$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *the questions.*
Each question carries 4 weightage.

25. Calculate two independent Frobenius series solutions of the equation $2xy'' + (x+1)y' + 3y = 0$.

26. Consider the differential equation $x(1-x)y'' + [p - (p+2)x]y' - py = 0$, where p is a constant. If p is not an integer, find the general solution near $n = 0$ in terms of hyper geometric functions.

27. Find the general solution of the system ; $\frac{dx}{dt} = 4x - 3y, \frac{dy}{dt} = 8x - 6.$

28. Prove that using Rodrigues formula $\int_{-1}^{1} P_n(x) P_n(x)\, dx = \frac{2}{2n+1}$.

(2 × 4 = 8 weightage)

C 23146 (Pages : 3) Name.....................................

Reg. No.................................

SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 2C 08—TOPOLOGY

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Define Discrete topology on X.
2. Let $X = \{a, b, c, d\}$, $\tau = \{\varnothing, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{X\}\}$ and $A = \{a, c, d\}$. Find adherent point Adh(A) = ?
3. Define derived set of A.
4. Let $X = \{a, b, c, d\}$; $\tau = \{\varnothing, \{a, b\}, \{a, b, c\}, \{a\}, \{a, c\}, X\}$ and $Y = \{b, c\}$. Find τ_Y.
5. Define quotient set.
6. What is meant by Tychonoffspace ?
7. Define uniformly converge on X.
8. Define compact subset.
9. Define hausdorff space.
10. What is meant by normal on X.
11. Define open set in a metric space.
12. What is meant by absolute property ?
13. Define topological space.
14. What is meant by usual topology on R ?

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Let X be a set and D be a family of subsets of X, then prove that there exists a unique topology τ on X, such that it is the smallest topology on X containing D.

16. Let (X, τ) be a topological space and $B \subset \tau$. Then prove that B is a base for τ if and only if for any $x \in X$ and any open set G containing x, there exists $B \in B$ such that $x \in B$ and $B \subset G$.

17. Prove that any family δ of subsets of X, there is a unique topology τ on X having δ as a sub-base. Further, every member of τ can be expressed as the union of sets each of which can be expressed as the intersection of finitely many members of δ.

18. Prove that for every subset $A \subset X$, $x_0 \in \bar{A}$ implies $(x_0) \in \overline{f(A)}$, then f is continuous at x_0.

19. Let $f : X \to Y$ be a continuous surjection, then prove that X is connected, so is Y.

20. Let X be a space and C be an connected subset of X (that is, C with the relative topology is a connected space), Suppose $C \subset A \cup B$ where A, B are mutually separated subsets of X. Then prove that either $C \subset A$ or $C \subset B$.

21. Let X_1, X_2 be topological spaces and $X = X_1 \times X_2$ with the product topology, then prove that X is connected.

22. Prove that a continuous bijection from a compact space onto a hausdorff space is a homeomorphism.

23. Prove that every map from a compact space into a T_2-space is closed.

24. Prove that every completely regular space is regular.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. a) Prove that a subset A of a space X is dense in X if and only if for every non-empty open subset B of X, $A \cap B \neq \emptyset$.

b) Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.

26. a) Prove that every closed surjective map is a quotient map.

    b) Prove that every second countable space is first countable.

27. a) Prove that every path connected space is connected.

    b) Let X be a Hausdorff space, $x \in X$ and F a compact subset of X containing x. Then prove that there exists open sets U, V such that $x \in U, F \subset V$ and $U \cap V = \emptyset$.

28. a) Prove that compact subset of a hausdorff space is closed.

    b) Prove that if (X, (Y, d), $\{f_n\}$) and $\{f_n\}$ converges to f uniformly, then each f_n is continuous, so is f.

(2 × 4 = 8 weightage)

C 23145 (Pages : 3) Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2022

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS—II

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Show that constant functions are measurable.

2. Show that every countable set has measure zero.

3. Prove that outer measure is translation invariant.

4. Give an example where strict inequality occurs in Fatou's Lemma.

5. Show that if f is a non-negative measurable function, then $f = 0$ if and only if $\int f\, dx = 0$.

6. Show that if f and g are measurable, $|f| \leq |g|\, a.e.$, and g is integrable, then f is integrable.

7. Show that BV $[a, b]$ is a vector space over the real numbers.

8. Show that the Lebesgue set of a function $f \in \mathrm{L}(a, b)$ contains any point at which f is continuous.

9. Show that Lebesgue measure m defined on M, the class of measurable sets of $\mathbb{R}$, is σ finite.

10. Show that $\mathrm{H}(\mathrm{R}) = \left\{\mathrm{E} : \mathrm{E} \subseteq \bigcup_{n=1}^{\infty} \mathrm{E}_n, \mathrm{E}_n \in \mathrm{R}\right\}$.

11. Give an example of a function which is continuous on $\mathbb{R}$ but not absolutely continuous.

12. Let f be a finite-valued monotone increasing function defined on (a, b), Show that $g(x) = f(x-)$ is left-continuous and monotone increasing on (a, b).

Turn over

13. Show that if $f \in L(a, b)$, its indefinite integral F is absolutely continuous on $[a, b]$.

14. Let μ be a complete measure on $[[X, \delta]]$ and $f : X \to \mathbb{R}$ is measurable, then μf^{-1} is a complete measure.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. State and prove countably additive property.

16. Show that there exists uncountable sets of zero measure.

17. Prove that there exists a non-measurable set.

18. Show that $\int_0^1 \frac{\sin t}{e^t - x} dt = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n^2+1}, -1 \le x \le 1.$

19. Prove that a function $f \in BV[a, b]$ if and only if f is the difference of two finite valued monotone increasing functions on $[a, b]$, where a and b are finite.

20. Let $[a, b]$ be a finite interval and let $f \in L(a, b)$ with indefinite integral F, then prove that $F' = f$ *a.e.* in $[a, b]$.

21. Let μ^* be the outer measure on $\mathcal{H}(\mathfrak{R})$ defined by on $\mathfrak{R}$, then prove that δ^* contains $\delta(\mathfrak{R})$, the σ ring generated by $\mathfrak{R}$.

22. Let μ be σ-finite measure and ν a σ-finite signed measure and let $\nu \ll \mu$; show that $\frac{d|\nu|}{d\mu} = \left|\frac{d\nu}{d\mu}\right| [\mu]$.

23. State and prove Jordan decomposition theorem.

24. Prove that every bounded linear functional F on C(I) can be written as $F = F^+ - F^-$, where F^+, F^- are positive linear functionals.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Let c be any real number and let f and g be real-valued measurable functions defined on the same measurable set E. Then prove that $f + c$, cf, $f + g$, $f - g$ and fg are also measurable.

26. State and prove Lebesgue Dominated convergence theorem.

27. If μ is σ finite measure on a ring $\mathfrak{R}$, then prove that it has a unique extension to the σ ring $\delta(\mathfrak{R})$.

28. State and prove Radon-Nikodym theorem.

(2 × 4 = 8 weightage)

C 23144 (Pages : 3) Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA—II

(2016 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Let $R = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : \det(A) \neq 0 \text{ and } a, b \in \mathbb{R} \right\}$ be a commutative ring with identity. Find the characteristic of R.

2. Is π^2 an algebraic over a field $\mathbb{Q}(\pi^3)$? Why ?

3. Find the algebraic closure of a field $\mathbb{Q}$?

4. Is $\mathbb{Z}_2[x]/\langle x^2 \rangle$ a field ? Why ?

5. Is 30° a constructible angle ? Why ?

6. Find the number of primitive 10^{th} roots of unity in $GF(23)$.

7. Find all automorphisms of $\mathbb{Q}(w, i)$ where w is cube root of unity.

8. Find the number of isomorphism of $\mathbb{Q}(\sqrt{3}, i)$ onto to a subfield of $\bar{\mathbb{Q}}$ leaving $\mathbb{Q}$ fixed.

9. Find the degree of the splitting field of $x^4 + x^2 + 1$ over $\mathbb{Q}$.

Turn over

10. Let $\mathbb{Q}\left(\sqrt{13}, \sqrt[3]{13}\right)$ be a finite separable extenstion of $\mathbb{Q}$. Find the primitive element of $\mathbb{Q}\left(\sqrt{13}, \sqrt[3]{13}\right)$.

11. Express $x_1^3 + x_2^3 + x_3^3$ as polynomials in the elementary symmetric functions in x_1, x_2, x_3.

12. Let $K = \mathbb{Q}\left(\sqrt[3]{5}, w\right)$ be a finite normal extension of $\mathbb{Q}$. Find $\lambda\left(\mathbb{Q}(w)\right)$.

13. If $K = \mathbb{Q}\left(\sqrt[4]{3}, i\right)$ is a splitting field of $x^4 - 3$ over $\mathbb{Q}$, prove or disprove : $G(K/\mathbb{Q})$ is solvable group.

14. Prove or disprove : the regular 60-gon is constructible.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Is $\langle x \rangle$ a maximal ideal in $\mathbb{Z}[x]$? Justify your answer.

16. Let α be a zero of $x^2 + 1$ over $\mathbb{Z}_3$. Find the multiplicative inverse of $1 + 2\alpha$ in $\mathbb{Z}_3(\alpha)$.

17. Determine the degree of the extension $\mathbb{Q}\left(\sqrt{1+\sqrt{-3}} + \sqrt{1-\sqrt{-3}}\right)$ over $\mathbb{Q}$

18. Give an example an infinite extension K of F such that K is an algebraic extension of F.

19. If F is any finite field, prove that for every positive integer n, there is an irreducible polynomial in $F[x]$ of degree n.

20. Find the conjugate of $\sqrt{2} + \sqrt[3]{3}$ over $\mathbb{Q}\left(\sqrt{2}\right)$.

21. Show that the Frobenius automorphism σ_p has order n.

22. Prove or disprove : the normal extension of a normal extension is normal.

23. Find $\Phi_{14}(x)$ over $\mathbb{Q}$.

24. If α and β are constructible real numbers, prove that $\alpha - \beta$ is constructible.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 weightage

25. (a) Let R be a finite commutative ring. Prove that every prime ideal in R is a maximal ideal.

    (b) Find all maximal ideals in $C[0,1]$, where $C[0,1]$ is the ring of all real valued continuous functions on $[0,1]$.

26. (a) Prove that a finite extension of a finite extension is finite.

    (b) Let p be a prime and $n \in \mathbb{N}$. If E and K are fields of order p^n, prove that $E \cong K$.

27. Prove that every field of characteristic zero is perfect.

28. Find the splitting field K of $x^4 - 4x^2 - 1 \in \mathbb{Q}[x]$. Compute the group of the polynomial over $\mathbb{Q}$ and exhibit the correspondence between the subgroups of $G(K/\mathbb{Q})$ and the intermediate fields.

(2 × 4 = 8 weightage)

C 4574 (Pages : 3) Name.....................................

Reg. No.....................................

SECOND SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION APRIL 2021

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS—II

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Show that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$ for any set B.

2. Show that every countable set has measure zero.

3. Prove that continuous functions are measurable.

4. Show that $\int_1^\infty \frac{dx}{x} = \infty$.

5. Show that if f is a non-negative measurable function, then $f = 0$ if and only if $\int f\, dx = 0$.

6. Give an example where strict inequality occurs in Fatou's Lemma.

7. Show that $BV[a, b]$ is a vector space over the real numbers.

8. Show that the Lebesgue set of a function $f \in L(a, b)$ contains any point at which f is continuous.

9. Let $f(x) = |x|$. Find the four derivatives at $x = 0$.

10. Let $f = g$ a.e. (μ), where μ is a complete measure. Show that g is measurable if f is measurable.

11. Let μ be a measure and let the measures v_1, v_2 be given by $v_1(E) = \mu(A \cap E), v_2(E) = \mu(B \cap E)$, where $\mu(A \cap B) = 0$ and $E, A, B \in S$. Show that $v_1 \perp v_2$.

12. Give an example of a function which is continuous on R but not absolutely continuous.

13. Show that if $f \in L(a, b)$, its indefinite integral F is absolutely continuous on $[a, b]$.

14. If $[a, b] \subset \bigcup_{i=1}^{n} (a_i, b_i)$, where $a_i, b_i, i = 1, 2, ..., n$, are bounded, then prove that $g(b) - g(a) \leq \sum_{i=1}^{n} (g(b_i) - g(a_i))$.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Prove that the outer measure of an interval equals its length.

16. Show that not every measurable set is a Borel set.

17. State and prove Fatou's lemma.

18. Show that $\int_0^1 \frac{x^{\frac{1}{3}}}{1-x} \log \frac{1}{x} \, dx = 9 \sum_{n=1}^{\infty} \frac{1}{(3n+1)^2}$.

19. Let $f \in BV[a, b]$, then prove that f is continuous except on a set which is atmost countable.

20. Let $[a, b]$ be a finite interval and let $f \in L[a, b]$ with indefinite integral F, then prove that $F' = f$ a.e. in $[a, b]$.

21. If μ is a σ finite measure on a ring R , then prove that it has a unique extension to the σ ring $S(R)$.

22. Let f and g be absolutely continuous on the finite interval $[a, b]$. Show that fg is absolutely continuous on $[a, b]$.

23. Prove that a union of sets positive with respect to a signed measure υ is a positive set.

24. Prove that Jordan decomposition of a signed measure is unique.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Prove that the set of all Lebesque measurable subsets of R is a σ-algebra.

26. State and prove Lebesgue Monotone convergence theorem.

27. State and prove Lebesgue differentiation theorem.

28. State and prove Hahn decomposition theorem.

(2 × 4 = 8 weightage)